THE SELECTIVE PROPERTIES

OF

COUPLED RADIO CIRCUITS

A Thesis submitted to the Graduate School of the University of Wisconsin in partial fulfillment of the requirements for the degree of Doctor of Philosophy

by

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Date June Seventh, 1920.

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This thesis having been approved in respect of form and mechanical execution is referred to you for judgment upon its substantial merit.

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CHAPTER 1.

Introduction

1. The Selective Properties of Alternating Current Circuits.

The impedance of most electric circuits to alternating currents depends upon the frequency of the alternating current. That is, if alternating voltages of the same amplitude but of different frequencies are impressed in a given circuit, the alternating circuits which follow in response may have widely differing values. Usually one or more frequencies can be found at which the current has a maximum value. This ability of an electric circuit to respond in differing degrees to voltages of different frequencies, is termed the selective property of the circuit.

This property is very useful in many fields, but it is particularly in the field of radio communication that this study of the selective properties of coupled circuits will be made. Selective circuits were used very early in the development of radio telegraphy, and they have become of increasing importance as the number of sending stations has increased. At the present time a receiving station must respond audibly only to signals within a very limited range of frequencies. Otherwise a mere jumble of tones would be heard in the receiver.

Several types of circuits have been proposed and tried in the effort to obtain greater selective properties. But most of these efforts have been less effective than they might have been because the underlying theory had never been fully worked out. There has been a real need for a systematic study of this property of electric circuits in order that selective circuits

might be more rationally designed.

2. Previous Studies of the Selective Properties of Circuits.

It was in response to this need that an analytical study of the properties of radio circuits was undertaken by Professor Edward Bennett, in 1917. The simple series receiving circuit was chosen as the logical starting point. This circuit consists of a tuned antenna circuit with a detector connected in series with the other elements of the circuit. A knowledge of the selective properties of this circuit is necessary before any great progress can be made in studying other circuits which have been proposed. For this circuit contains the antenna; and it is such questions as the effect of antenna dimensions upon the energies received from signals and from strays, that cause the most confusion among radio engineers. Therefore, in addition to the purely selective properties of series circuits, the beginning study was concerned with the ability of antennas to abstract energy from electromagnetic waves, or it was concerned with the abstractive properties of antenna circuits.

As a result of this study, a paper[1] on "The Abstractive and Selective Properties of Radio Antenna Circuits", was prepared by Professor Bennett and this paper has been used as a foundation for the present study of the selective properties of coupled circuits. Further discussion of this paper appears in the later sections.

[1] To appear in the Proceedings, and Transactions, of the American Institute of Electrical Engineer, 1920.

Other publications have touched the selective properties of circuits rather incidentally. Dr. George W. Pierce in his book[1], "Electric Oscillations and Electric Waves", which has been published since this thesis was prepared, gives some attention to some of the questions considered in this thesis. He discusses the adjustments of two coupled circuits which will result in the largest delivery of power to the secondary circuit, and arrives at equations (35) and (39) as presented in this thesis. His conclusions however, are somewhat less general than those stated herein. He also defines a "Ratio of Interference", which serves the same purpose as the "Selective Coefficient", defined in the paper dealing with simple circuits and used in the present paper. But the conclusions stated in his book, page 200, differ markedly from the conclusions reached in this study. The difference is due to the fact that his conclusions are drawn from numerical calculations for coupled circuits containing very high resistance. In such circuits the change in frequency produces relatively little effect upon the currents, and the conclusions drawn do not hold for circuits with low resistance such as are considered in this paper.

Other investigations of the selective properties of circuits that have been described, have dealt usually with special circuits and have been largely experimental.

3. The Abstractive and Selective Properties of Radio Antenna Circuits.

Since the paper by Professor Bennett with the above title

[1]McGraw Hill Co., 1920.

is to be taken as a foundation for the study of coupled circuits, a brief statement of some of the relations brot out in that paper, is necessary. Only those relations which are to be used in this study, will be mentioned.

It was there pointed out that "the function of a receiving circuit is not simply to deliver to a detecting device energy abstracted from impinging waves. Its function is to selectively abstract energy from the impinging waves and to deliver it to the detecting device. The ultimate comparison of the relative merits of two antennas for receiving purposes under the conditions to be met in commercial operation, must be a comparison of the amounts of energy delivered to the detectors from the desired correspondent when the strengths of the interfering signals or noises have been reduced to the same intensity in the two cases".

The selective coefficient of a receiving circuit against an interfering source, was defined as the ratio between the energy received from the correspondent's station and the energy received from the interfering source, when the peak electric intensities of the two waves are equal at the antenna.

Interfering sources were classified and expressions were derived for the selective coefficients of simple circuits against the more important types of these interferent sources. Some of these expressions will be presented as needed later in this paper. One important fact may be noted. The selective coefficient was found in every case to be expressible in terms of the impressed frequency and the time constant of the circuit. The antenna dimensions did not appear directly in these ex-

pressions, and therefore, in subsequent investigations such as the present, one is justified in making any assumptions that are convenient regarding the antenna dimensions, provided the argument is dealing with selective properties alone.

It was pointed out that the power delivered to the detector is a maximum when the resistance of the detector (R_d) is equal to the sum of all other resistance in the circuit, that is, equal to the sum of the radiation resistance (R_r) and the wasteful resistance (R_w). This equality was assumed to exist thruout the study of simple circuits.

Assuming then an ideal antenna in which $R_w = o$, it was shown that the power delivered to the detector by this ideal antenna is equal to $E_1^2/4R_r$ where E_1 represents the effective voltage induced in the antenna. And the important result was then brot out, that $E_1^2/4R_r$ is independent of the antenna dimensions. That is, the maximum power which an antenna can abstract from given waves, and deliver to a detector, is independent of the antenna dimensions provided the wasteful resistance can be made equal to zero, or more generally, provided the ratio $R_r/(R_w + R_r)$ is kept constant. This ratio was represented by $1/k$ and was called the "Abstractive Efficiency" of the system since the power delivered to this ideal antenna, is equal to this ratio. In order to make the abstractive efficiency equal to 100%, the wasteful resistance must be zero. From this standpoint the only advantage a large antenna has over a small antenna for receiving purposes, is the better abstractive efficiency which usually is obtained with the large antenna, at an increased cost of course. If the same money is invested in

each this advantage tends to disappear.

The power abstracted from the electromagnetic waves by an actual antenna or even by the ideal antenna in which R_w is zero, is a very small part of the power radiated by the sending station. This is due to two facts. First, the power radiated from the sending station is radiated in all directions in the hemisphere above the plane of the earth, or the directive property of the system is poor. And second, a very large part of the power radiated is lost by absorption between the sending station and the hemisphere upon which the receiving station is located, or the transmitting property is poor.

In order to separate these two losses an expression derived analytically was given for the power streaming across the hemisphere upon which the receiving station is located. The power which may be abstracted by the ideal antenna in which R_w is zero, divided by the power streaming across the hemisphere was called the "directive efficiency" of the system. And the power streaming across the hemisphere divided in turn by the power radiated by the sending station was called the "transmission efficiency" of the system. The actual power streaming across the hemisphere may be different from the power calculated by the expression which was given, but the efficiencies as thus defined were used in such ways that the conclusions are not in error on this account. The actual power delivered to a detector by any antenna is equal to the actual power radiated by the sending station multiplied successively by the transmission efficiency, the directive efficiency and the transmission efficiency.

The transmission efficiency depends upon the frequency and the distance between stations in a way which is known only approximately. Equation (22) gives the transmission efficiency as calculated from the experimental data of Dr. Austin of the Bureau of Standards.

$$T_r = \epsilon^{-\frac{.003 r}{\sqrt{\lambda}}} \qquad (r \text{ and } \lambda \text{ in km}) \tag{22}$$

Other relations which were brot out in the paper dealing with simple circuits will be discussed in later sections of this paper[1] as they are needed.

4. Object and Method of the present Paper.

In the previous paper the abstractive efficiency of antenna circuits and the selective coefficients against interference of various types, were defined and discussed quantitatively. The object of the study was to determine the frequencies and the circuit proportions which would make the circuit most highly responsive to the correspondent's signal, and least responsive to interferent sources.

The present paper has for its object the extension of this treatment to systems in which two tuned magnetically coupled circuits are used in receiving. As in the previous paper the problem is to determine the frequency and the receiving circuit constants which will make this type of circuit most highly responsive to the correspondent's signal and least responsive to interferent sources.

[1] In such cases the paper by Professor Bennett will be referred to simply as the previous paper since the two are intended to form part of a series. For the same reason the equations will be numbered consecutively in the two papers.

The discussion is limited to the case in which undamped waves are used for signalling, but similar arguments would apply roughly to the use of damped waves. Magnetic coupling between circuits is the only type which is herein considered. Other forms of coupling and special circuit combinations can be studied more easily after the properties of this type of circuit have been discussed.

The method of arriving at the best relations between the circuits constants may be briefly outlined. First the desirable properties of the system are listed, then each one in turn is considered independently and the relations between the circuit constants which secure this property are derived. This treatment brings out several sets of desired relations between the circuit constants; the final step is to compare these sets and to effect a compromise between the conflicting requirements.

5. Natural Division of the Discussion Into Two Parts.

The constants of the system for which the adjustments are to be found are the operating frequency, or the wave length, and the constants of the receiving circuit. Now the choice of the operating frequency should be based on a consideration of the properties and requirements of the whole system--the sending station, the transmitting medium, and the receiving station. On the other hand, the choice of the constants of the receiving circuit to operate at any specified frequency is purely a local problem,--that is, it should be based on a consideration of the properties of the receiving circuit alone. It is best to keep these two problems separate, and for this purpose the discussion will be divided into two parts, namely

(a) Considerations which should govern the design of the receiving circuit for a given operating frequency.

(b) Considerations which should govern the choice of the operating frequency.

The two problems are taken in this order since an intelligent choice of operating frequency will depend partly upon the characteristics of the receiving station as brot out in the first problem. In dealing with the choice of the receiving circuit constants the operating frequency will be regarded as a fixed constant.

We proceed in chapter II to a discussion of this first problem.

6. Acknowledgements.

This study of the selective properties of coupled circuits is a part of the general study of the selective properties of circuits as planned by Professor Bennett. He has guided this study thru regular and frequent conferences in which Mr. L. J. Peters also took part.

CHAPTER II

Considerations which should Govern the Design of a Coupled Circuit Receiving Station for any given Operating Frequency

7. Desirable Properties of the Receiving Circuit.

The properties which are desirable or necessary in the receiving circuit may be listed in a form convenient for quantitative discussion.

(a) The power delivered to the detector from signal waves of a given electric intensity should be a maximum.

(b) The selective coefficients of the circuit against interferent sources should be maxima.

(c) The time constant of the circuit should not be increased beyond certain fixed limits.

The best receiving circuit is considered to be that one which is most highly responsive to signals and least responsive to interference. The listing of properties (a) and (b) follows directly from this statement since by the selective coefficient of a circuit against an interferent voltage is meant the ratio between the energy received in the detector from the signal and the energy received from the interferent voltage. The maximum electric intensities or the peak voltages induced in the antenna in the two cases are assumed equal. Two types of interferent voltages are considered, waves from detuned sending stations, and strays of the type corresponding to unidirectional impressed voltages.

Of these two properties (a) and (b) the second is regarded as being of far more importance than the first. That is, the absolute value of the power delivered to the detector from the

signal waves is of less importance than the ratio between this power and the power delivered from interferent sources, or the selective coefficient. This is true because of the present possibilities in the way of power amplification.

The third property, a limited time constant is listed because of two facts brot out in the discussion of series circuits. First, in order to have a period of silence between tones in the receiver, the time constant of the circuit must be limited to about one fifth of the interval between signals.

Second, due to the small but unavoidable variations in frequency of the sending station and in the resonant frequency of a receiving circuit, it is impossible to hold the two frequencies exactly equal. For a large share of the signals there is a small detuning which causes the reactance in the receiving circuit to be different from zero. For a circuit of given inductance the signal current is increased very little by reducing the resistance below the value at which it is equal to this small reactance. Thus for simple receiving circuits an independent limit is set beyond which nothing is gained by increasing 2L/R or T_c. This limit may not be the same in coupled receiving circuits as in simple circuits, but it is apparent that a limit will be set in the same way,

The time constant of a coupled circuit receiving station is not so obviously defined nor so easily calculated as for simple circuits. But by analogy, it may be defined as the time necessary after voltage is applied for the current in the detector to reach 63% of its steady state value, (r.m.s. values over 1 cycle are referred to.) With this definition the time

constant of coupled circuits is limited by the first consideration to the same values as in the case of simple series circuits. These values are about .01 sec. for undamped wave reception at the rate of 30 words per minute, and .0004 sec. for reception of damped waves, 1000 wave trains per second.

A discussion of the time constant and the selective coefficient against strays involves a discussion of the transient currents in coupled oscillatory circuits. A general analytical treatment is very involved since the damping factors and the frequencies of the transient currents must be determined from an equation of the fourth degree.

The method adopted in this paper consists in first discussing for the general case, the steady state conditions in the coupled circuits. From this study of the steady state conditions a tentative arrangement of circuit constants is obtained and by numerical calculations for the transient currents in this tentative arrangement, and in variations from it, equations more or less empirical are derived relating to the time constant and to the selective coefficients to strays.

8. Expressions for the Steady State Currents and Power in Coupled Receiving Circuits.

Let Fig. 11 represent the circuits to be considered in

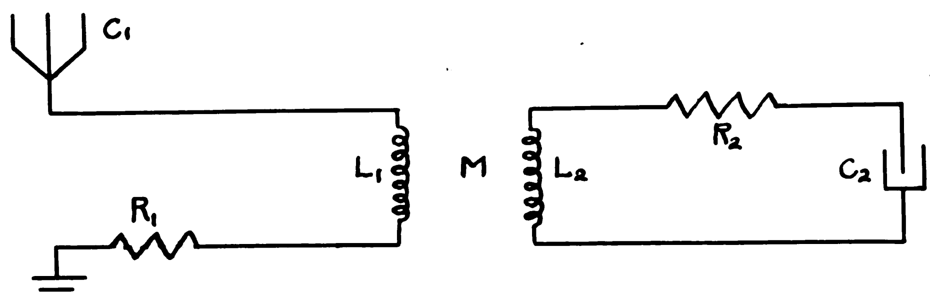


Fig. 11.

this paper. In this diagram R_1 must include the radiation resistance of the antenna, and R_2 must include the detector resistance. It is assumed that the detecting device can be represented by a series resistance. The voltage impressed in the circuit acts between the antenna and ground. Its effect is the same as if an alternator were connected in the circuit at any point. Another assumption is that the antenna circuit can be represented by a circuit containing lumped inductance and capacity.

The independent circuit constants appearing in Fig. 11 are seven in number, R_1, R_2, L_1, L_2, C_1, C_2, and M. But if the discussion is carried on in terms of these constants, the equations become very long and complicated. Certain expressions appear repeatedly in these equations and it is much more convenient to use abbreviations for these expressions, and to think of these abbreviations as the real independent constants. Among these new constants are of course the reactances of the circuits. Also in simple oscillatory circuits 2L/R is often called the time constant of the circuit, for reasons which appear in the study of the transient currents in such circuits. In this treatment of coupled circuits the same abbreviation will be used altho the physical meaning of the term no longer holds.

The seven constants defined by the equations below are seen by inspection to be independent; that is, any one of them may be varied at will while the others are held constant. And since seven is the total number of independent constants in two magnetically coupled circuits, these constants completely fix the circuits.

R_1

R_2

X_1 represents $2\pi f L_1 - 1/2\pi f C_1 = \omega L_1 - 1/\omega C_1$

X_2 " $2\pi f L_2 - 1/2\pi f C_2 = \omega L_2 - 1/\omega C_2$

T_{c1} " $2L_1/R_1$

T_{c2} " $2L_2/R_2$

X_m " $2\pi f M = \omega M$

In these and all following equations the letter f without a subscript will be used to indicate the signal frequency. And ω will be used to represent 2π times the signal frequency, or $2\pi f$. In writing the equations for the currents and voltages in the coupled circuits, the method of complex numbers will be used. Complex quantities will be represented by bold faced capitols.

The fundamental equations are obtained from Kirchoff's law that the sum of the impressed and the counter voltages in any circuit is equal to zero. Applying this law to each circuit in turn gives the two equations:

$$\mathbf{E}_1 - R_1\mathbf{I}_1 - jX_1\mathbf{I}_1 - jX_m\mathbf{I}_2 = 0$$

$$-R_2\mathbf{I}_2 - jX_2\mathbf{I}_2 - jX_m\mathbf{I}_1 = 0 \qquad (31)$$

These two equations solved algebraically for $\mathbf{I}_1$ and $\mathbf{I}_2$, give

$$\mathbf{I}_1 = \frac{\mathbf{E}_1(R_2 + jX_2)}{R_1R_2 - X_1X_2 + X_m^2 + j(R_1X_2 + R_2X_1)}$$

$$\mathbf{I}_2 = \frac{-j\mathbf{E}_1X_m}{R_1R_2 - X_1X_2 + X_m^2 + j(R_1X_2 + R_2X_1)} \qquad (32)$$

These equations when interpreted according to the usual rules for the complex notation, give both the value and the relative

phase positions of the currents. Since this discussion has to deal mainly with the power and energy relations in the circuits, the expressions for the absolute values, as given in equations (33), are more convenient.

$$I_1 = \frac{E_1(R_1^2+X_1^2)^{1/2}}{\left[X_m^4+2X_m^2(R_1R_2-X_1X_2)+(R_1^2+X_1^2)(R_2^2+X_2^2)\right]^{1/2}}$$

(33)

$$I_2 = \frac{E_1 X_m}{\left[X_m^4+2X_m^2(R_1R_2-X_1X_2)+(R_1^2+X_1^2)(R_2^2+X_2^2)\right]^{1/2}}$$

The power expended in the primary circuit is of no especial interest in this discussion. The power delivered to the detector is the important quantity. But this power can be made practically equal to the total power in the secondary circuit. Calculations have shown that tuning coils in which 2L/R does not exceed 0.1, do not require prohibitive amounts of copper. Thus in secondary circuits in which $2L_2/R_2$ does not exceed .01, R_2 can be made 10 times the resistance of the coil alone, and therefore 90% of the power delivered to the secondary is expended in the detector. No distinction will be made between the power delivered to the secondary, and the power expended in the detector.

The power, P_2, which represents $R_2 I_2^2$ may be expressed as

$$P_2 = \frac{R_2 E_1^2 X_m^2}{X_m^4+2X_m^2(R_1R_2-X_1X_2)+(R_1^2+X_1^2)(R_2^2+X_2^2)} \qquad (34)$$

9. Relations between the Circuit Constants which give Maximum steady state Power to the Detector.

In equation (34) the power in the secondary circuit is

expressed in terms of five of the independent variables, R_1, R_2, X_1, X_2, and X_m. The power is evidently independent of T_{c1} and T_{c2} since they do not appear in the equation. The problem is to find the relations between these five constants which result in the maximum power delivered to the detector. The most obvious way of finding these relations is to equate the partial derivatives of P to zero.

Let us first seek the adjustment of the primary reactance, X_1, which will result in the largest delivery of power to the secondary. Equating $\frac{\delta P}{\delta X_1}$ to zero, and solving the equation gives

$$X_1 \text{ should equal } \frac{X_2 X_m^2}{R_2^2 + X_2^2} \tag{35}$$

This equation gives the adjustment of the primary reactance, which will result in the largest power delivery to the secondary. It holds for any values of the other constants.

It should be pointed out that X_2 may be either positive or negative in sign depending upon whether the inductive reactance or capacity reactance of the secondary is the larger. And to satisfy equation (35) the primary reactance, X_1, must have the same sign. That is, both net reactances must be alike, either both inductive or both condensive.

The adjustment of the primary reactance is such as to bring the primary current into phase with the impressed voltage as may be shown by transforming the expression for the primary current in equation (32) to the form (32a)

$$I_1 = \frac{E_1}{R_1 + R_2\frac{X_m^2}{R_2^2+X_2^2} + j\left(X_1 - X_2\frac{X_m^2}{R_2^2+X_2^2}\right)} \tag{32a}$$

The following independent proof is available that this is

the adjustment of X_1 which gives the maximum secondary power. The secondary impedance and the mutual reactance are constant while X_1 is being varied, and thus both the voltage induced in the secondary, and the current which flows in the secondary are proportional to the primary current. And from (32a) it is evident that this adjustment of X_1 makes I_1 a maximum, since it makes the absolute value of the denominator a minimum.

Let the best adjustment of the mutual reactance, X_m, be considered next. By setting $\delta P_2/\delta X_m = 0$ and solving the equation, the following condition is obtained

$$X_m \text{ should equal } \sqrt{(R_1^2+X_1^2)(R_2^2+X_2^2)} \tag{36}$$

For any possible combination of R_1, R_2, X_1, and X_2, this adjustment gives the largest value of P_2 which can be obtained thru adjustment of X_m alone.

Now it is possible to adjust X_1, and X_m so that equations (35) and (36) are simultaneously satisfied. The adjustments of X_1 and X_m which give this result are found most readily by treating equations (35) and (36) as simultaneous equations and solving for X_1 and X_m.

$$\frac{X_1}{X_2} = \frac{R_1}{R_2} = \frac{\sqrt{R_1^2+X_1^2}}{\sqrt{R_2^2+X_2^2}} \tag{37}$$

$$X_m^2 = \frac{R_1}{R_2}(R_2^2+X_2^2). \qquad Or\ X_m^2 = \frac{X_1}{X_2}(R_2^2+X_2^2) = \frac{R_2}{R_1}(R_1^2+X_1^2) = \frac{X_2}{X_1}(R_1^2+X_1^2) \tag{38}$$

Equation (38) is given in three equivalent forms for future reference.

These adjustments give the largest secondary power which it is possible to obtain by adjusting only these two constants, X_1 and X_m. Substituting these relations in equation (34)

gives (34a)

$$P_2 = \frac{E_1^2}{4R_1} \tag{34a}$$

in which R_1 is the only circuit constant appearing.

This expression represents the maximum power obtainable in the secondary circuit since P_2 can be increased further only by decreasing R_1. Adopting the notation of the previous paper, namely, letting R_r represent the radiation resistance and R_w the wasteful resistance of the primary, and further letting $\frac{1}{k}$ represent the ratio $\frac{R_r}{R_r + R_w}$, we have

$$P_2 = \frac{1}{k} \cdot \frac{E_1^2}{4R_r} \tag{34b}$$

Now it has been shown (section 13 of the previous paper) that $\frac{E_1^2}{4R_r}$ is independent of the constants or dimensions of the receiving antenna, or

$$\frac{E_1^2}{4R_r} = P_s D_e T_r$$

in which P_s represents power radiated from sending station

D_e represents directive efficiency of the system

T_r represents transmission efficiency

In the previous paper, $\frac{1}{k}$ is termed the abstractive efficiency A_e of the antenna.

$$A_e = \frac{1}{k} = \frac{R_r}{R_r + R_w}$$

The only way then to further increase P_2 is to increase the abstractive efficiency, by decreasing the wasteful resistance of the primary circuit.

A_e should be as large as possible.

Referring back to equation (34a) it is evident that the maximum power which can be obtained in the secondary circuit is independent of the secondary resistance and reactance since R_2 and X_2 have cancelled out of the equation. This statement brings out a result which may be rather surprising. It has of

course been realized that R_2 could be chosen more or less arbitrarily and the coupling could be correspondingly adjusted so that P_2 remained the same. But it may not have been so generally recognized the <u>no tuning adjustment need be made in the secondary circuit in order to get maximum secondary power</u>.

This fact suggests the possibility that the same maximum secondary power, $P_2 = E_1^2/4R_1$, can be obtained by adjusting any two of the four constants, X_1 , X_2 , X_m and R_2 , and leaving the other two constants entirely free or arbitrary. Trial shows this supposition to be correct except in special cases. The best adjustments of X_2 and R_2 for any conditions are obtained from $\frac{\delta P_2}{\delta X_2} = 0$, and $\frac{\delta P_2}{\delta R_2} = 0$, respectively. These adjustments are

$$X_2 \text{ should equal } \frac{X_1 X_m^2}{R_1^2 + X_1^2} \tag{39}$$

$$R_2 \text{ should equal } X_2^2 + \frac{X_m^4 - 2X_m^2 X_1 X_2}{R_1^2 + X_1^2} \tag{40}$$

And then by trial it is found that when any two of the four equations, (35), (36), (39), or (40), hold simultaneously, the two may be reduced to the relations (37) and (38). Thus the same maximum power is obtainable since the adjustments are equivalent.

<u>Equations (37) and (38) may be regarded therefore as general relations which must exist between the circuit constants in order to make the power delivered to the secondary, a maximum.</u> And this maximum power can be obtained by any adjustment of the circuit constants X_1 , X_2 , X_m and R_2 which satisfies these two equations.

Before leaving this subject, two special cases will be

briefly pointed out, in which it is not possible to obtain this maximum power by adjusting a particular pair of the four constants named.

Equation (37) contains the same condition that was noted in equation (35), i.e. X_1 and X_2 must have the same sign. If they do not have the same sign, it is evident that equation (37) can never be satisfied by any possible adjustments of the pair of constants X_m and R_2. This is one special case in which the supposition that any pair could be used, is not correct.

Another special case arises when X_m^2/R_1R_2 is less than 1. For equation (38) can be put in the form $X_1 = R_1\sqrt{\frac{X_m^2}{R_1R_2} - 1}$ and thus for this condition the equation is imaginary. The reactances X_1 and X_2 do not appear under the radical, therefore it is impossible to satisfy this equation by adjusting this pair of constants.

All such exceptions are taken care of when the conclusions are stated as follows:

To obtain maximum power in the secondary circuit two conditions are necessary.

(a) <u>The abstractive efficiency of the antenna circuit should be as high as possible, or the primary wasteful resistance should be low.</u>

(b) <u>The circuit constants should be adjusted so that equations (37) and (38) are both satisfied.</u>

10. Relations between the Circuit Constants which give Maximum Steady State Selective Coefficient against Sustained Detuned Frequencies.

As stated in a previous paragraph the steady state selec-

tive coefficient against detuned frequencies will be discussed before other selective coefficients, since calculations for the steady state are more easily made than for the transient states. The steady state selective coefficient of a receiving circuit against sustained detuned frequencies is defined as the ratio between the steady state energy (or power) received in the detector from the correspondent's signal, and the energy (or power) received from a detuned station. The voltages induced in the antenna in the two cases are to be equal.

The currents, powers, and reactances at the two frequencies will be distinguished by using the additional subscript "d" to indicate the frequency of the detuned station. When no subscript appears the signal frequency is to be understood. Detuning of the order of 1% or 2% is considered, since it is desired to eliminate all signals outside of a very small range of frequencies. With this difference in frequency the mutual reactances at the two frequencies differ by only 1% or 2% and the distinction may be dropped. But X_1 the primary reactance at signal frequency, may be quite different from X_{1d}, the primary reactance at the detuned frequency. These distinctions cannot be dropped.

The expression for the selective coefficient may be obtained from the definition as P_2/P_{2d} or I_2^2/I_{2d}^2, which gives

$$S_c = \frac{X_m^4 + 2X_m^2(R_1R_2 - X_{1d}X_{2d}) + (R_1^2 + X_{1d}^2)(R_2^2 + X_{2d}^2)}{X_m^4 + 2X_m^2(R_1R_2 - X_1X_2) + (R_1^2 + X_1^2)(R_2^2 + X_2^2)} \qquad (41)$$

Equation (41) contains the quantities X_{1d} and X_{2d}, which represent the net reactances at the detuned frequencies. In order to keep the discussion in terms of the seven independent circuit constants which were chosen in section (23), these

reactances should be expressed in terms of those constants.

If the detuned frequency is expressed as $(1+p)f$, in which p represents the decimal parts detuning, the reactance X_{1d} may be written

$$X_{1d} = \omega L_1(1+p) - \frac{1}{\omega C_1(1+p)}$$

But from the definition of X_1,

$$X_1 = \omega L_1 - \frac{1}{\omega C_1} \quad , \text{ or } \quad \frac{1}{\omega C_1} = \omega L_1 - X_1$$

and then substituting this and also substituting $(1-p)$ for $1/(1+p)$ which is justifiable since p is of the order .02, gives the form.

$$X_{1d} = \omega L_1(1+p) - \omega L_1(1-p) + X_1(1-p) = 2p\omega L_1 + X_1(1-p).$$

And finally since $2L_1 = R_1 T_{c1}$, and $X_1(1-p)$ is very nearly equal to X_1,

$$X_{1d} = p\omega R_1 T_{c1} + X_1$$
$$X_{2d} = p\omega R_2 T_{c2} + X_2 \qquad (42)$$

Now these expressions might be substituted into equation (41) to give an equation for the selective coefficient, in terms of the independent circuit constants. But the equation obtained is so long and unwieldy that the attempt to equate the partial derivatives to zero will not be made. An easier argument is sought.

Let the adjustments of X_1 and X_2 be considered first while the other constants remain fixed. Now consider for a moment the ordinary resonance curve for coupled circuits, that is, the curve between current in the secondary of two fixed circuits, as ordinates, and the impressed frequency as abscissae. Such curves for coupled circuits are shown in Fig. 12, and in Fig. 17,

section 14. There may be either one or two peaks.

It is very evident that in order to use the selective properties of such circuits to advantage, the circuit constants must be so adjusted that one of the current peaks occurs at the signal frequency. If this is not done, the selective properties of the circuit may be a disadvantage rather than an advantage. Or in other words the reactances should be so adjusted that the current or power delivered to the detector has the maximum value.

The adjustments of the reactances which give the largest secondary power under any set of conditions are given by equations (35) and (39). To find the values of X_1 and X_2 which satisfy both of these equations at once, the two equations are solved simultaneously for X_1 and X_2. As a result three pairs of values or roots of the equations are obtained. The first two pairs are

$$\begin{aligned} X_1 &= \pm R_1 \sqrt{\frac{X_m^2}{R_1 R_2} - 1} \\ X_2 &= \pm R_2 \sqrt{\frac{X_m^2}{R_1 R_2} - 1} \end{aligned} \tag{43a}$$

Now X_1 and X_2 must have the same sign to satisfy equation (35) but either sign may be chosen. Let the positive sign be used here. It is sufficient to give the investigation for one pair of these values since the other pair gives very similar results in the numerical calculations.

These values of X_1 and X_2 satisfy equations (37) and (38) therefore these adjustments make the power delivered to the detector from the signal waves, equal to the maximum possible power.

It will be convenient to introduce into these equations a

new independent circuit constant J, which is defined by the equation.

$$J = \frac{X_m^2}{R_1 R_2} \qquad (44)$$

This constant hereafter replaces X_m as an independent circuit constant; i.e. the value of X_m is fixed by equation (44) and it ceases to be independent.

The constant J is made directly proportional to the second power of X_m since in all expressions for power or energy the mutual reactance enters with even powers. The physical arguments which appear later are simpler in terms of J as defined here than they would be in terms of the square root of this value. Otherwise a constant varying directly as X_m would be preferred.

This constant will be termed the "coupling index" since it indicates or fixes the mutual reactance.

Substituting this index in equation (43a) and using only the positive sign before the radicals, gives

$$\begin{aligned} X_1 &= R_1\sqrt{J-1} \\ X_2 &= R_2\sqrt{J-1} \end{aligned} \qquad (43)$$

The substitution of the values of X_1 and X_2 from equations (43) in equations (42) gives

$$\begin{aligned} X_{1d} &= R_1\left(p\omega T_{c1} + \sqrt{J-1}\right) \\ X_{2d} &= R_2\left(p\omega T_{c2} + \sqrt{J-1}\right) \end{aligned} \qquad (42a)$$

Now substituting equations (42a) and (43) in (41) and reducing to the simplest form gives an expression for the selective coefficient, which applies when the reactances are adjusted to the values indicated in equation (43).

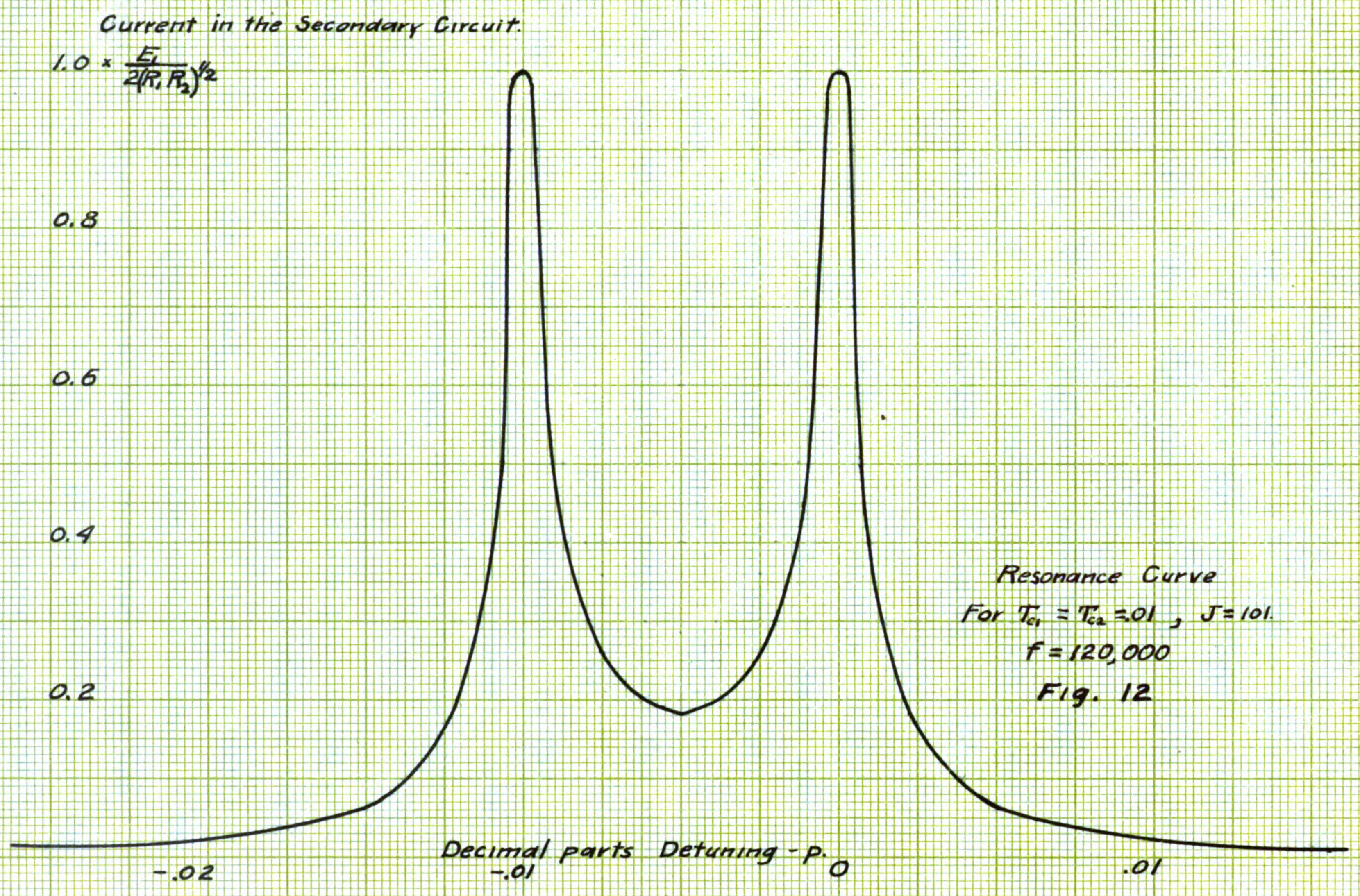
$$S_c = \frac{\rho^4\omega^4T_{c1}^2T_{c2}^2 + \rho^2\omega^2(T_{c1}-T_{c2})^2 + 2\rho^3\omega^3T_{c1}T_{c2}(T_{c1}+T_{c2})\sqrt{J-1} + \rho^2\omega^2(T_{c1}+T_{c2})(J-1)}{4J} + 1 \quad (41a)$$

It is seen that R_1 and R_2 cancel out of this equation and only T_{c1}, T_{c2}, and J remain.

Now let J also vary and it is evident upon inspection that as J decreases from some large value towards unity, the numerator because of its large constant term, will decrease proportionately slower than the denominator. Therefore the best value of J is the lowest value.

A numerical calculation will make this more plain however so in the first columns of Table XI, are given some numerical values of S_c and J calculated from this equation for a circuit in which T_{c1} and T_{c2} each equal .01, and ω is assumed as 200,000.

Values of the selective coefficient are given for both positive and negative values of p, that is, for detuned frequencies both higher and lower than the signal frequency. The coefficients differ for the two cases except when J= 1, because the resonance curve for these coupled circuits under the conditions assumed, has two peaks for all values of J except for J = 1. At this value the two peaks merge. The resonance curve for J = 101 which is presented in Fig. 12 shows the two peaks. From this Figure it is seen that the curve is not symmetrical with respect to the signal frequency. It is possible for the signal current to have the value of one peak and the interferent current to have the value of the other peak, in which case the selective coefficient would be approximately 1. The two peaks are exactly equal only when T_{c1} and T_{c2} are equal.



Resonance Curve

For $T_{c_1} = T_{c_2} = .01$, $J = 101.$

$f = 120,000$

Fig. 12

Comparing now the values of the selective coefficient as given in Table XI for different values of J, it is seen that S_c increases quite rapidly as J decreases to 1. (Equation (43) cannot be satisfied if J is less than 1) These figures bear out the statement previously made that the best value of J is the lowest value.

And at this value, J = 1, the reactances become zero from equation (43). These are the adjustments then which appear best when the equations (35) and (39) are satisfied by the first pair of roots.

The other pair of values of X_1 and X_2 which satisfy equations (35) and (39) are the pair $X_1 = 0$ and $X_2 = 0$. Note that this pair of roots satisfy equations (35) and (39) for any values of J. They should be distinguished from the special values, $X_1 = X_2 = 0$, to which the first pair of roots reduce for only one value of the coupling index, that is, for J = 1.

This third pair of roots do not satisfy equations (37) and (38). Therefore they do not give the maximum secondary power, except of course at J = 1, where they coincide with the other pairs of values.

If the reactances of coupled circuits are reduced to zero at the signal frequency and resonance curves are plotted for these conditions, at the signal frequency is found the minimum current between the two peaks whenever the coupling index J is greater than 1. When J is equal to or less than 1, the two peaks merge and then the peak is found at the signal frequency. This adjustment of X_1 and X_2 is to be considered therefore only when J is equal to, or less than 1.

When $X_1 = 0$ and $X_2 = 0$ equation (42) gives

$$X_{1d} = p\omega R_1 T_{c1}$$

$$X_{2d} = p\omega R_2 T_{c2} \qquad (42b)$$

And substituting these values in equation (41) gives an expression for the selective coefficient when the reactances X_1 and X_2 are adjusted to zero.

$$S_c = \frac{p^4\omega^4 T_{c1}^2 T_{c2}^2 + p^2\omega^2(T_{c1}-T_{c2})^2 - 2p^2\omega^2 T_{c1}T_{c2}(J-1)}{(J+1)^2} + 1 \qquad (41b)$$

From inspection of this equation it is again evident that the selective coefficient increases as J decreases, but numerical values will make this more striking. In Table XI the last column contains some calculated coefficients for the same circuit constants assumed in the previous calculations.

TABLE XI

Selective coefficients for two coupled circuits with various values of the coupling index.

$T_{c1} = T_{c2} = .01$ sec. $\qquad \omega = 200{,}000.$

J	For $X_1 = R_1\sqrt{J-1}$, $X_2 = R_2\sqrt{J-1}$.		For $X_1 = X_2 = 0$
	$p = .02$	$p = -.02$	$p = \pm.02$
101	14400	1600	
10	73700	46400	21000
2	352000	288000	252000
1	640000	640000	640000
0.5	Adjustment impossible		1,140,000
0	"	"	2,560,000

The tabulated values show that for this assumed adjustment $X_1 = X_2 = 0$, the value of J can be carried to zero, and the selective coefficient increases steadily as J decreases. But at $J = 0$, no power is delivered to the secondary, so it is evident a compromise will be necessary in choosing J. At the present stage of the discussion it seems evident that J should be equal to or less than 1. And for these values of the coupling index the values of the reactances are $X_1 = X_2 = 0$.

Thus no further attention is necessary to adjustments of the reactances different from zero. It will be assumed hereafter that these constants are fixed.

The constants R_1 and R_2 cancelled out of the equations for the selective coefficient so the coefficient is independent of them.

The constants yet to be considered are T_{c1} and T_{c2}. Since the adjustment $X_1 = X_2 = 0$ is decided upon above, the equation which should be used for the selective coefficient is equation (41b).

From this equation it is evident that the selective coefficient increases indefinitely as either T_{c1} or T_{c2} increase, and therefore they should be as large as possible.

In equation (41b) the first term in the numerator is much larger than the other terms for most circuits to be considered later, so dropping the other terms the equation becomes

$$S_c = \frac{p^4 \omega^4 T_{c1}^2 T_{c2}^2}{(J+1)^2} \tag{41c}$$

From this expression is obtained the more definite conclusion that the selective coefficient increases as the <u>second powers</u> of T_{c1} and T_{c2}.

The conclusions for this section may now be stated. To secure a maximum steady state selective coefficient against detuned frequencies the circuit constants should be adjusted as follows:

The coupling index J should be equal to or less than 1.

The net reactances at the signal frequency, X_1 and X_2 should be zero.

The constants T_{c1} and T_{c2} should be as large as possible.

The resistances R_1 and R_2 may be chosen arbitrarily.

11. Relations between the Circuit Constants which will give a Time Constant of a Specified Value.

The time constant of the coupled circuits has been defined as the time necessary for the secondary current to build up to 63% of its final value. For the reasons pointed out, this time constant should be limited to about .01 sec. for reception of undamped waves. The purpose in this section is to learn what conditions must be imposed upon the circuit constants in order to keep the time constant within the limits desired.

The time constant is to be calculated for special circuits arranged to meet the conditions laid down in the preceding sections. These conditions fix X_1 and X_2 as zero. J is limited in range to values in the neighborhood of 1, say between 0.2 and 1.0 but it still is regarded as an arbitrary constant. The other arbitrary constants are R_1, R_2, T_{c1} and T_{c2}.

The time constant of the coupled circuits as defined above will be represented by T_c. First let the relation between T_c and R_1 and R_2 be considered. The following proposition may be stated: If T_{c1}, T_{c2} and J are held constant and $X_1 = X_2 = 0$, the time constant T_c is independent of R_1 and R_2.

To prove this we make use of the differential equations which express the relations between currents and voltages both for the transient period and for the steady state. Kirchoff's law states that the sum of all impressed and counter voltages in a circuit is zero. Applying this law to the primary circuit gives,

$$e_1 - R_1 i_1 - L_1 \frac{di_1}{dt} - \frac{q_1}{C_1} - M\frac{di_2}{dt} = 0 \tag{44}$$

Differentiating once with respect to t gives (44a) since $\frac{dq_1}{dt} = i_1$

$$L_1 \frac{d^2 i_1}{dt^2} + R_1 \frac{di_1}{dt} + \frac{i_1}{C_1} + M\frac{d^2 i_2}{dt^2} = \frac{de_1}{dt} \tag{44a}$$

And is a similar way for the secondary circuit, (44b) applies. The impressed voltage is zero.

$$L_2 \frac{d^2 i_2}{dt^2} + R_2 \frac{di_2}{dt} + \frac{i_2}{C_2} + M\frac{d^2 i_1}{dt^2} = 0 \tag{44b}$$

Assume that the currents i_1 and i_2 which satisfy these equations have been found.

Now let the secondary resistance be made n times as large, equal to nR_2. T_{c2} and X_2 are to remain constant, therefore the secondary inductance must become equal to nL_2 and the reciprocal of the capacity must become n/C_2. The coupling index J, which represents $X_m^2/R_1 R_2$ is also assumed fixed so the mutual inductance must become $\sqrt{n}$ times as large, equal to $\sqrt{n}\,M$. Then represent the currents which flow in these circuits as i_1' and i_2' and the differential equations may be written for these conditions.

$$L_1 \frac{d^2 i_1'}{dt^2} + R_1 \frac{di_1'}{dt} + \frac{i_1'}{C_1} + \sqrt{n}\,M\frac{d^2 i_2'}{dt^2} = \frac{de_1}{dt} \tag{45a}$$

$$nL_2\frac{d^2 i_2'}{dt^2} + nR_2\frac{d\,i_2'}{dt} + \frac{n\,i_2'}{C_2} + \sqrt{n}\,M\frac{d^2 i_1'}{dt^2} = 0 \tag{45b}$$

Now the currents i_1' and i_2' which satisfy these equations may be found by assuming that $i_1' = i_1$ and $i_2' = i_2/\sqrt{n}$. For on making these substitutions in (45a), it reduces to (44a) indentically, and therefore is satisfied since (44a) was assumed to be satisfied, and on substituting in (45b) and dividing each term by $\sqrt{n}$ it reduces indentically to (44b). Therefore these substitutions satisfy both (45a) and (45b). The primary current then is the same as before and the secondary current is $1/\sqrt{n}$ times as large as before the change. The only effect of making the secondary resistance n times as large is to make the secondary current at each instant $1/\sqrt{n}$ times as large. The power dissipated each instant is the same as before, since $R_2 i_2^2 = nR_2 (i_2/\sqrt{n})^2$ and likewise the energy stored at each instant, $\frac{1}{2}L i^2 + \frac{1}{2}\cdot\frac{q^2}{C}$, in each circuit is the same as before. And the time constant has not been changed since the secondary current at each instant has been changed by a fixed ratio.

A similar argument may be carried thru for changes in the primary resistance. If R_1 is increased n times, the applied voltage must be multiplied by $\sqrt{n}$ in order to keep the power the same. Otherwise the argument is parallel. The secondary current remains unchanged and the primary current is reduced to $1/\sqrt{n}$ times the former value.

Thus T_c is independent of R_1 and R_2 and in each numerical examples the resistances may be set at any convenient values provided T_{c1}, T_{c2} and J remain as chosen for that example.

The problem therefore is to find the conditions which must

be imposed upon T_{c1}, T_{c2} and J to limit T_c to the value desired. This will be done by calculation of T_c for special cases.

Case 1. For the purpose of these calculations a numerical value must be assigned to the frequency. The value 120,000 will be used in the following calculations. In working thru the calculations, it becomes evident that the relations derived will hold for any frequency. The primary constants are made appromimately equal to those of the Darien antenna, T_{c2} is chosen as .02 and J as I. The constants are::

$$L_1 = L_2 = 176\,\mu.h.\,(10^{-6}\text{henries}), \quad C_1 = C_2 = .01\,\mu.f.,$$
$$R_1 = 10.75, \quad R_2 = .0176, \quad T_{c1} = .00003, \quad M = .57\,\mu.h. \tag{46}$$

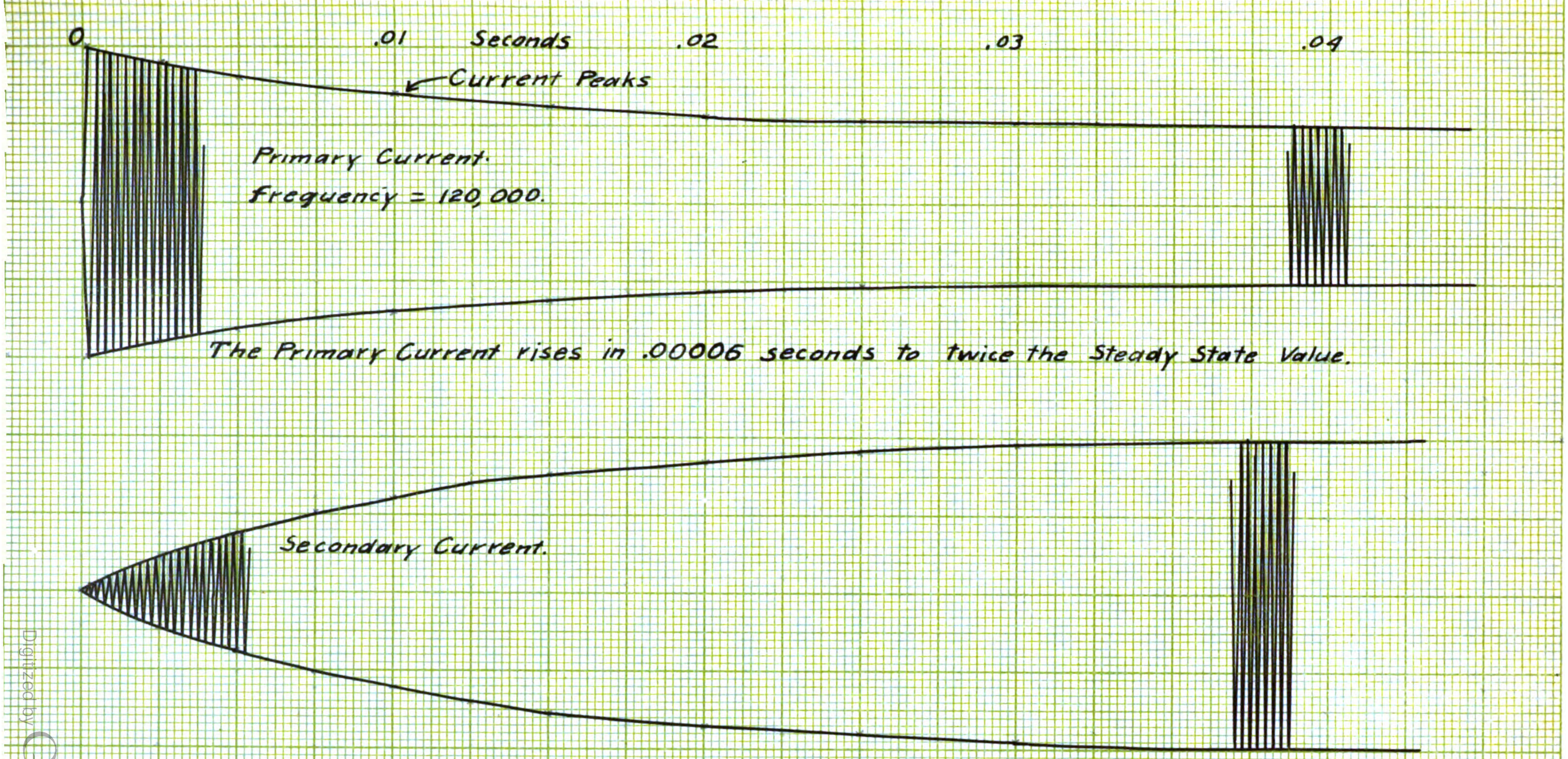
When the voltage $E_1 \sin\omega t$. is impressed on the primary the currents are given very closely by the simplified equations.[1] $(\omega = 2\pi \cdot 120{,}000.)$

$$i_1 = .066\, E_1 \sin\omega t\left(1 - 2\varepsilon^{-30494t} + \varepsilon^{-96t}\right)$$
$$i_2 = -1.625\cos\omega t\left(1 + .003\varepsilon^{-30494t} - 1.003\varepsilon^{-96t}\right) \tag{47}$$

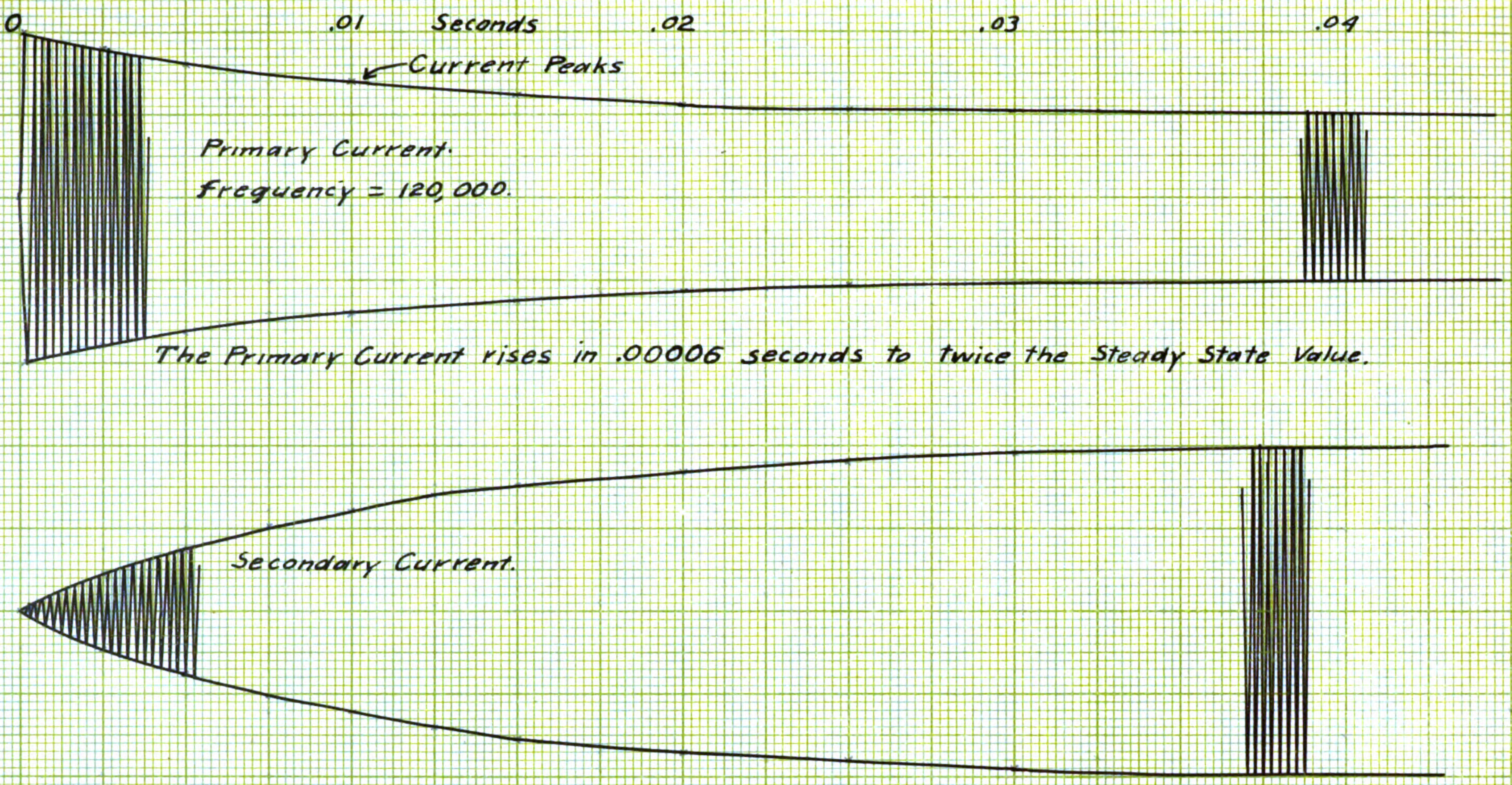
In Fig. 13 are shown these currents for .05 sec. after the voltage is impressed. The primary current rises in about .00005 sec. to twice the steady state value since the secondary current and the reaction from it are still negligible.

Then as the secondary current builds up the reaction from it reduces the primary current until both reach their steady state values. The time necessary for the secondary current to

[1] Formulas and discussions which are very helpful in calculating these currents are given by V. Bush, Proc. Inst. of Radio Engineers, Vol. 5 pages, 363-382; by Pernot, Bulletin University of California, and by others.



Starting Currents in Coupled Circuits.
From Equations (47).
$T_{c_1} = .00003$, $T_{c_2} = .02$, $J = 1$.
Fig. 13



Starting Currents in Coupled Circuits.
From Equations (47).
$T_{c_1} = .00003$, $T_{c_2} = .02$, $J = 1$.
Fig. 13

reach 63% of its final value is seen from the graph to be .01 seconds, or $T_c = .01$.

The voltage acting in the secondary circuit is proportional to the primary current and therefore starts out at twice the steady state value. The curve showing the primary current also represents the voltage induced in the secondary. As a result of this high voltage during the transient period, it is seen that the secondary current reaches 63% of its steady state value in .01 sec. while it would take .02 sec. if a steady state voltage were impressed in this circuit. The voltage which may be regarded as active in increasing the secondary current at any instant, is the difference between the induced voltage and the resistance drop. (The reactance is zero). This difference at any instant is about twice as great in the case of the coupled circuits as it would be with a steady voltage impressed on the secondary. Therefore it is natural that the secondary current should build up twice as fast in the case of the coupled circuits.

And furthermore if the conditions were such that the primary current rose quickly to 3 times the final value and then decreased, one would expect that the secondary current would build up in about 1/3 of .02 sec. Or T_c is approximately equal to T_{c2} divided by this ratio between the initial and the final current.

The ratio between the initial peak of the primary current and the final value may be easily calculated. The peak is E_1/R_1 since the reaction from the secondary is negligible. The steady state value is obtained from (33) by substituting $X_1 = X_2 = 0$,

and $X_m^2 = JR_1R_2$. It reduces to $E_1/R_1(1+J)$. Dividing E_1/R_1 by this expression gives the ratio between the currents which is $(1+J)$. Thus one might reasonably expect that the time necessary for the secondary current to build up would depend upon the coupling index and could be represented approximately by the formula,

$$T_c = \frac{T_{c2}}{1+J} \qquad (48a)$$

for this circuit in which the primary time constant is so short as to be relatively inappreciable.

Case II. The constants chosen for the next numerical calculation are such as to provide a test of formula (48a), just proposed. The antenna circuit is left the same as before, the coupling index is reduced, let J = 0.4, and T_{c2} is made equal to .01 sec. The constants are:

$$L_1 = L_2 = 176\,\mu.h., \qquad C_1 = C_2 = .01\,\mu.f.$$
$$R_1 = 10.75 \qquad R_2 = .0352. \qquad M = .5\,\mu.h. \qquad (49)$$

In this circuit the two currents are given very closely by the simplified expressions

$$i_1 = .094\,E_1 \sin\omega t \cdot \left(1 - 1.4\,\varepsilon^{-30500t} + .4\,\varepsilon^{-140t}\right)$$
$$i_2 = -1.04\,E_1 \cos\omega t \cdot \left(1 + .003\,\varepsilon^{-30500t} - 1.003\,\varepsilon^{-140t}\right) \qquad (50)$$

The value of T_c may be obtained by plotting a graph of the secondary current or from the fact that the only exponential term of any importance is the term ε^{-140t}. The time constant is the value of t which reduces the exponent to -1. This is $\frac{1}{140}$ = .007 sec. Applying now the formula (48a) gives $T_c = .01/1.4 = .007$, which is the correct value

Case III. This case differs from those preceding, in that T_{c1}, the time constant of the primary circuit, is made equal to T_{c2} and is no longer inappreciable in comparison. Let $T_{c1} = T_{c2} = .01$ and $J = 1$. The circuit constants chosen are:

$$L_1 = L_2 = 242\,\mu.h., \qquad C_1 = C_2 = 0073\,\mu.f.$$
$$R_1 = R_2 = .0484\,, \qquad M = .53\,\mu.h. \tag{51}$$

When $E_1 \sin\omega t.$ is suddenly applied to the primary the currents are given by

$$i_1 = 14.6\,E_1 \sin\omega t\left[1 - \varepsilon^{-100t}(\cos 100t - \sin 100t)\right]$$
$$i_2 = -14.6\,E_1 \cos\omega t\left[1 - \varepsilon^{-100t}(\cos 100t + \sin 100t)\right] \tag{52}$$

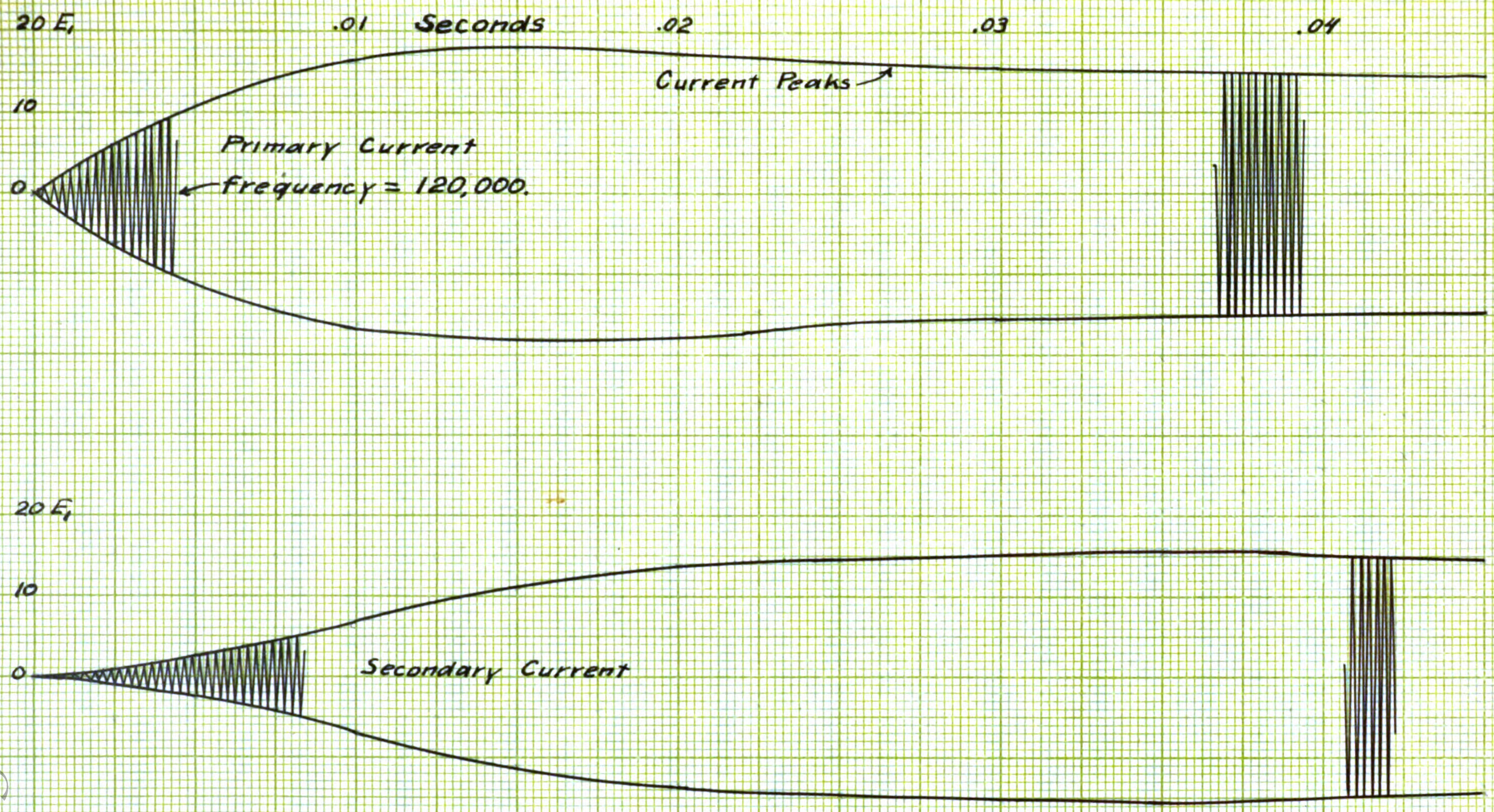
These currents are plotted in Fig. 14 for .05 sec. after application of the voltage. From the graph T_c may be measured and is about .012 sec. Thus as might have been expected the conclusion drawn from Case I in which T_{c1} was very short do not hold derectly for this circuit and the formula (48a) will have to be modified.

The results of the three calculations are grouped in Table XII.

TABLE XII

T_{c1}	T_{c2}	J	T_c
.00003	.02	1.	.01
.00003	.01	0.4	.007
.01	.01	1.	.012

Now a simple formula is desired to express approximately the relations between T_{c1}, T_{c2}, J and T_c as shown in Table XII. Such a simple formula is given by modifying (48a)



Starting Currents in Coupled Circuits
From Equations (52)
$T_{c_1} = T_{c_2} = .01, \quad J = 1.$
Fig 14.

(48)

This formula (48) may be taken as expressing the relation which is necessary between T_{c1}, T_{c2} and J, in order to limit the time constant of the circuit in the way desired. The equation is empirical and only approximate, but on the other hand the limits placed on the time constant are only approximate, and it is believed the formula is sufficiently close for the purpose. The formula expresses a fixed relation and therefore is accepted immediately and assumed to apply thru out the discussion hereafter.

12. Relations between the Circuit Constants which will give a Maximum Selective Coefficient against "Strays".

In the previous paper (section 17) "stray" voltages were classified according to wave shape. Of the types named there, only one--the unidirectional sustained voltage was considered important. The damped oscillating stray voltages does not in general have the same frequency as the signal voltage and therefore the selective coefficient against this type of stray corresponds roughly to the selective coefficient against detuned signal waves. At least, the best adjustment against detuned signals is a good adjustment against this type of strays. The impulse type of stray which is assumed to last only during a small fraction of a cycle, is eliminated from the discussions since it is felt to be relatively unimportant.

The type of stray to be considered then is represented by unidirectional voltages which exist for a few hundredths of a second. The energy received from these strays by the antenna

becomes approximately equal to $\frac{1}{2}C_1E_1^2$. This is the energy which would be received if a battery voltage were suddenly applied, or a charge were suddenly released on the antenna.

The selective coefficient of the receiving circuit against unidirectional stray voltages has been defined as the ratio between the energy received in the detector, from a signal during the interval of excitation T_e , and the energy received from a stray. The peak voltages induced in the antenna by the two sources are to be the same.

With a simple series circuit the selective coefficient against this type of stray has been given, section 17 of the previous paper, as

S (simple series circuit against strays) $= 2\pi^2 b T_e T_{c_1} f^2$ (16)

in which T_e is the interval during which energy is received from the signal waves. The coefficient b is the ratio between the average power received from the signal during this interval and the steady state power which would result if the sending key were held down. It may be assumed constant for the conditions discussed in this paper, and approximately equal to 0.8.

In the case of coupled circuits the expression for the selective coefficient differs from equation (16) because of the different way in which the energy from the stray divides between the resistances of the circuit. In the series circuit the total energy received from a stray, $\frac{1}{2}C_1E_1^2$, divides half to the detector and half to the radiation and wasteful resistances provided the detector resistance is equal to $R_r + R_w$. In the coupled circuits this energy may divide in any ratie, depending upon the coupling and other constants. Thus the response to

strays in coupled circuits may be quite different from the response in simple circuits. The response to signals is the same in both cases if the same antenna is used since $P_2 = E_1^2/4R_1$ for each.

These facts suggest the method which is used below for calculating S_c for coupled circuits. First, for a stray voltage acting on coupled circuits, calculate the fraction of the total energy which reaches the detector in the secondary. Represent this fraction by 1/A. Then because of the facts stated above A/2 is the ratio between the selective coefficient for the coupled receiving circuits, and the selective coefficient for the antenna circuit alone, after a detector with a resistance equal to $R_r + R_w$ has been inserted in it. This last selective coefficient is given by equation 16. Then

$$S_c \text{ (coupled circuits against strays)} = \frac{A}{2} \times 2\pi^2 b T_a \frac{T_{e1}}{2} f^2 \qquad (53)$$

In this formula, T_{e1}, which represents 2L/R for the primary circuit without the detector, must be divided by 2, since the insertion of a detector doubles the primary resistance. This formula is of little use in drawing general conclusions since the factor A depends in unknown ways upon the circuit constants. It is intended only as an aid in the numerical calculations of selective coefficients for special cases by means of which the general relations are to be found.

In section (10) the reactances X_1 and X_2 were fixed at zero. It was assumed that no requirement would appear in later sections to conflict with these which appeared in that section. The question whether the selective coefficient against strays can be increased by departing from this relation, now will be

briefly considered.

Assume that the coupling index J is kept equal to 1. Then any departure from the relation, $X_1 = X_2 = 0$, will have two effects. First it will greatly reduce the energy received by the antenna from the signal waves. On the other hand the energy received from the strays will remain constant, and equal to $\frac{1}{2} C_1 E_1^2$. This effect tends to greatly reduce the selective coefficient. The second effect is to change the ratios in which the energies divide between the two circuits. These ratios change both for the energy received from signals and that from strays. In circuits so highly oscillatory as these circuits, the damping of the transient oscillations is very slight, and the laws of the circuit for the damped oscillations may be expected to correspond somewhat to the laws for undamped oscillations. That is, the ratios between the energy dissipated in the two circuits may be expected to vary in somewhat the same way in the two cases. At least the statement may be made that, since the first effect will so greatly reduce the selective coefficient it is very improbable that the second effect can be made to counteract and overcome the first. Therefore no appreciable increase in the selective coefficient against strays can be expected from letting X_1 and X_2 differ from zero. It is more likely that the coefficient will be decreased. Therefore the relation $X_1 = X_2 = 0$ will still be assumed to hold.

The argument based on the differential equations in the preceding section, showed that the energy and power relations in the two coupled circuits are independent of R_1 and R_2 provided T_{c1}, T_{c2} and J are fixed. Thus in equation (53) the

coefficient A which depends upon the division of energy between the primary and secondary, is independent of R_1 and R_2. These constants do not appear at all then in equation (53) and S_c is independent of them.

The selective coefficient is fixed therefore, by the values given to the remaining independent circuit constants which are T_{c1}, T_{c2}, and J. Or since equation (48) fixes T_{c2} when T_c, T_{c1} and J are given, the selective coefficient is fixed when these three constants are given. The constant T_c is a constant in which we are directly interested, so it will be convenient from here on to consider it as one of the independent constants and then T_{c2} is fixed by equation (48) and ceases to be independent. The three independent circuit constants in which variations are yet to be considered, are T_{c1}, T_c and J. They will be taken up in the order named. The effect of variations in T_{c1}, will be studied by means of numerical calculations for special cases in all of which $J = 1$, $T_c = .01$ and the frequency is 120,000.

Case 1. Choose T_{c1} so that $T_{c1} = T_{c2}$. From equation (48) this condition gives $T_{c1} = T_{c2} = .0083$. Let the circuit constants be the same as for Case III section 26, except that the resistances are increased 20% in order to reduce T_c to .01.

$$L_1 = L_2 = 242 \mu.h., \qquad C_1 = C_2 = .0073 \mu.f.$$

$$R_1 = R_2 = .058, \qquad M = .72 \mu.h. \tag{54}$$

In this circuit $T_{c1} = T_{c2}$ and $\dfrac{1}{L_1C_1} = \dfrac{1}{L_2C_2} = \omega^2$

This last relation is equivalent to $X_1 = X_2 = 0$. Under these special conditions the exact general expressions for the transient currents can be derived. More definite conclusions can be drawn in some of the later arguments if the calculation is

carried out in the general form, so in Appendix A, this work is outlined. It is there shown that the exact expressions for the currents due to a unidirectional stray can be simplified by introducing some very close approximations, and can be reduced to the equations:

$$
\begin{aligned}
i_1 &= \frac{E_1}{\omega L_1} \sin \omega t \cdot \varepsilon^{-t/T_{c1}} \cos \frac{\sqrt{J}\, t}{T_{c1}} \\
i_2 &= -\sqrt{\frac{L_1}{L_2}} \cdot \frac{E_1}{\omega L_1} \cos \omega t \cdot \varepsilon^{-t/T_{c1}} \sin \frac{\sqrt{J}\, t}{T_{c1}}
\end{aligned} \tag{55a}
$$

The power (averaged over one cycle) expended at any time in each circuit is given by

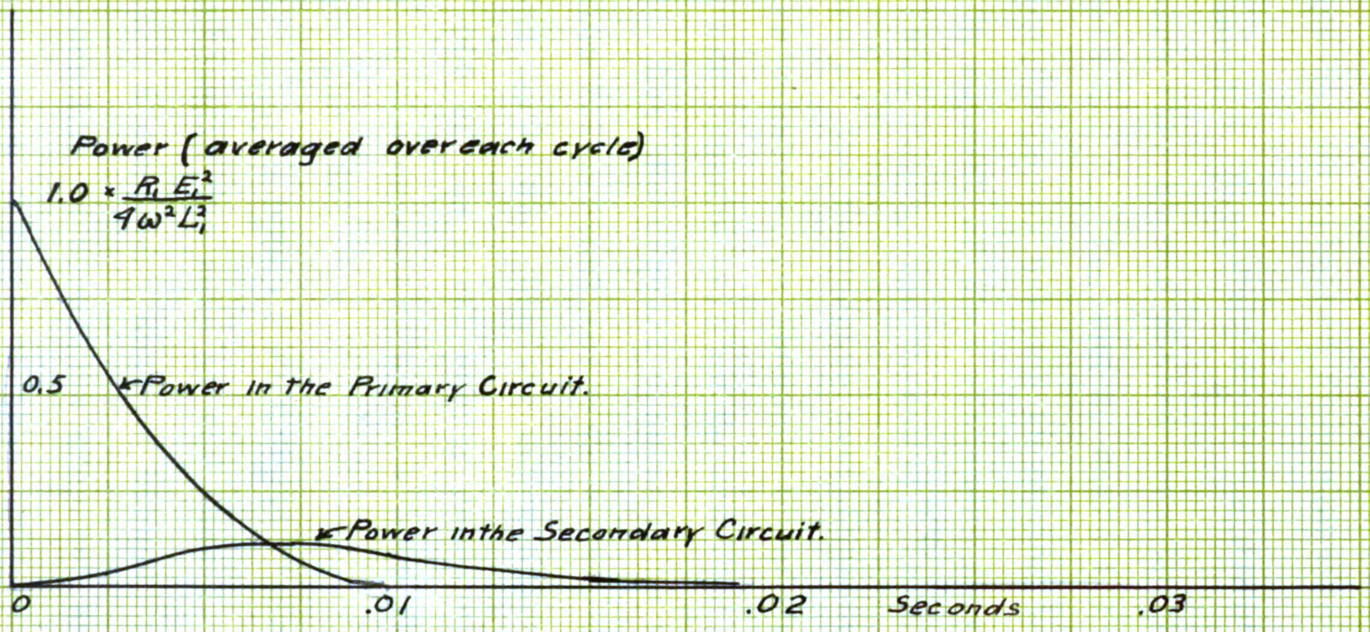
$$
\begin{aligned}
P_1 &= \frac{R_1 E_1^2}{4 \omega^2 L_1^2}\, \varepsilon^{-2t/T_{c1}} \left(1 + \cos \frac{2\sqrt{J}\, t}{T_{c1}}\right) \\
P_2 &= \frac{R_1 E_1^2}{4 \omega^2 L_1^2}\, \varepsilon^{-2t/T_{c1}} \left(1 - \cos \frac{2\sqrt{J}\, t}{T_{c1}}\right)
\end{aligned} \tag{55b}
$$

And the total energy expended in each circuit is given by,

$$
\begin{aligned}
W_1 &= \frac{C_1 E_1^2 (2 + J)}{4\,(1 + J)} \\
W_2 &= \frac{C_1 E_1^2 J}{4 (1 + J)}
\end{aligned} \tag{55c}
$$

When J = 1, as in this case, the power in each circuit is shown in the curves Fig. 15. From these curves or from similar curves drawn to show the currents as given in the equations above, the growth and decay of the currents can be traced.

The areas under the power curves Fig. 15 represent the total energy expended in each circuit and by comparison of the areas it is found that 3/4 of the total energy is expended in the primary and 1/4 in the secondary. The same result is obtained by substituting J = 1, in (55c). Then $W_1 = \frac{3}{8} C_1 E_1^2$, and $W_2 = \frac{1}{8} C_1 E_1^2 =$ one fourth of the total energy received by the antenna, which is $\frac{1}{2} C_1\ E_1^2$.



Power delivered to Coupled Circuits by a suddenly impressed sustained Voltage.

For $T_{c_1} = T_{c_2} = .0083$, $L_1C_1 = L_2C_2$, $J=1$.

Fig. 15

The factor A then in equation (53) is equal to 4 for this set of conditions and the value of the selective coefficient against strays may be obtained from equation(53).

$$S_c(T_{c_1}=.0083) = \frac{4}{2} \times 2\pi^2 b T_e \frac{.0083}{2} f^2 = .016\pi^2 b T_e f^2. \tag{56}$$

In section (11) it was stated that the relation which was derived between T_c, T_{c_1}, T_{c_2}, and J, would hold at frequencies other than the frequency assumed in making the numerical calculations. In support of that statement it may be observed that in equation (55a) for i_2, the factor $\varepsilon^{-t/T_{c_1}} \sin \frac{\sqrt{J}\,t}{T_{c_1}}$ which governs the amplitudes of the successive oscillations and therefore gives the time constant, is independent of f, the frequency of oscillation, and depends only on T_{c_1} and J.

Case II. Let the circuit constants be exactly as given for Case I, section (11), since there $T_c = .01$, $J = 1$, and $f = 120{,}000$. $T_{c_1} = .00003$ and $T_{c_2} = .02$. Similar calculations for this circuit show that 1/1000 of the total energy received from the stray, is delivered to the detector, or A = 1000 for this case.
Then from equation (53)

$$S_c(T_{c_1}=.00003) = \frac{1000}{2} \times 2\pi^2 b T_e \frac{.00003}{2} f^2 = .015\pi^2 b T_e f^2. \tag{57}$$

which is practically the same as for Case I.

Case III. A dead beat antenna circuit with the detector in a tuned secondary is sometines proposed as a method of eliminating strays, so a circuit of this type is now considered. Calculations were made for a circuit in which $T_c = .01$, $J = 1$, and $f = 120{,}000$ as before, $T_{c_1} = 1.3 \times 10^{-6}$ and $T_{c_2} = .02$.
About 1/6000 of the total energy was found to be expended in the secondary, or for this case, A = 6000. And from equation (53)

$$S_c(T_{c_1} = 1.3\times10^{-6}) = \frac{6000}{2} \times 2\pi^2 b T_e \frac{1.3\times10^{-6}}{2} f^2 = .004\pi^2 b T_e f^2 \tag{58}$$

This coefficient is somewhat less than for the other cases. So no gain is made by having a dead beat antenna circuit.

A comparison of the coefficients for the three cases, as given in equations (56) (57) and (58), show that the changes in S_c are so small for large variations in T_{c_1} that for the purpose here they may be neglected.

S_c is independent of T_{c_1} when T_c and J are constant. (59)

This conclusion can be checked by the following approximate physical argument. Let the value of T_{c_1} be controlled by changing the wasteful resistance in the primary. The antenna dimensions then are constant. From equation (34a) the energy received by the detector from the signal waves varies as $1/R_1$. The total energy received from a sustained unidirectional electromotive force is constant $(\frac{1}{2}C_1E_1^2)$ since it depends only on the antenna dimensions. This energy from the stray is dissipated in the primary at a rate proportional to R_1. The interval during which the disturbance lasts in the primary and during which a voltage is induced in the secondary, is proportional to $1/R_1$. The energy transferred to the secondary varies roughly as this interval and is therefore proportional to $1/R_1$. Thus both for strays and for signals the response varies as $1/R_1$ or the selective coefficient is constant.

Next consider the effect of variations in T_c. Since S_c is approximately independent of T_{c_1}, this constant may be varied at will duing the argument without causing any change in S_c. And it will be of aid to keep T_{c_1} always equal to T_{c_2} for then equations (55a) (55b) and (55c) will apply.

Then the energy received from a sustained unidirectional

electromotive force divides as shown in equation (55c). If J is kept equal to 1, one fourth of the energy from the stray always reaches the detector in the secondary. And if the antenna dimensions are constant, the response to stray voltages is constant.

But under these same conditions the energy received from the signal voltages varies as $1/R_1$ from equation (34c) below, or as T_{c1}. T_{c1} varies as T_c when $T_{c1} = T_{c2}$ so finally the energy received from the signal varies as T_c. The selective coefficient against strays varies as T_c also since the energy received from the strays is constant.

S_c (against strays) varies as T_c when J is constant (60)

The last constant to consider is the coupling index, J.

Again make use of the fact that the selective coefficient is nearly independent of T_{c1}, and assume that T_{c1} will be kept equal to T_{c2}. Then equations (55c) apply and the energy received in the detector from strays varies as $J/(1+J)$.

The power or energy received from the signal waves, varies with the coupling index in a way expressed by equation (34c) which is obtained from (34) by substituing $X_1 = X_2 = 0$, and $X_m^2 = JR_1 R_2$

$$P_2 = \frac{E_1^2 J}{R_1(1+J)^2} \tag{34c}$$

Under the conditions assumed, T_c is to remain constant and T_{c1} is to be kept equal to T_{c2}. Under these conditions T_{c1} must vary as $(1+J)$ from equation (48), and since the antenna dimensions were assumed fixed, R_1 must vary as $1/T_{c1}$ or as $1/(1+J)$. Consideration of this fact in connection with equation (34c) shows that the power or energy received from the signal varies

as J/(1+J).

Thus the energy received from the signal depends upon J in the same way that the energy from the stray does. Therefore the selective coefficient is independent of J.

S_c (against strays) is independent of J. (61)

The selective coefficient against strays then is independent of T_{c1}, and of J, while it varies directly as T_c. These relations must be regarded as approximate. But this section is concerned with factors of the order of 10, or larger, since increases in the selective coefficient of that order or larger are sought. This being the case these relations are satisfactory. They may be put in the form of equation (60a)

$$S_c \,(\text{against strays}) = H\,T_c \tag{60a}$$

in which the coefficient H may be evaluated by putting in the known values for one of the numerical examples which have been worked out, such as in Case I of this section. Accordingly a comparison of equation (56) with (60a) gives

$$H\,T_c = .016\,\pi^2 b T_e f^2 \qquad \text{Or } H = 16\,bT_e f^2$$

and

$$S \,(\text{against strays}) = 16\,bT_e f^2 T_c \tag{62}$$

This equation serves for coupled circuits as equation (16) of the previous paper does for simple series circuits. From this equation or from (59), (60), and (61) the conditions which give a maximum selective coefficient against undirectional strays are quite obvious and may be stated as

The time constant T_c should be as large as possible.

The other circuit constants may be left as fixed by other requirements.

There is a definite limit upon the value of the time constant T_c, so the possibility of greatly increasing the selective coefficient against strays seems small.

13. Choice of Circuit Constants based upon the Relations obtained in the preceding Sections.

The seven independent circuit constants in terms of which the preceding discussion has been presented, were listed with their defining equations in section 8 as R_1, R_2, T_{c1}, T_{c2}, X_1, X_2, and X_m. In equation (44) the coupling index J, was introduced and in the later sections it has replaced X_m as one of independent circuit constants since the arguments could be made more direct thereby. When J is considered independent, X_m is fixed by equation (44). Likewise in the section just preceding, the constant T_c was used as an independent constant to replace T_{c2}. When T_c is independent, T_{c2} is fixed by equation (48). The conclusions in this section will be stated in terms of the list as amended, that is in terms of R_1, R_2, T_{c1}, T_c, X_1, X_2, and J. It has been important at all times, and still is, to keep definitely in mind the list of constants which are considered as the independent constants. For example, variations in R_1 produce very different effects if L_1, C_1, M, etc. are to remain constant, than they do when T_{c1}, X_1, and J are to remain constant. The independent constants should be so chosen as to make the arguments most direct, and the relations most simple.

The adjustments of these constants, found in any preceding section to be desirable, have been set down usually without comparing them with the adjustments which have been found desirable in other sections. The purpose of this section is to

review all of the adjustments and to effect compromises between conflicting requirements.

In each of the preceding sections it has been found that R_2 could be left entirely arbitrary. This is fortunate since now the secondary circuit can be built to suit the requirements of a given detector without sacrificing any other desirable end.

The reactances X_1 and X_2 should be each adjusted to zero in the interest of maximum steady state selective coefficient against sustained detuned frequencies. It has been shown that this adjustment is at least as good as any other, when the selective coefficient against strays is considered. No conflicting requirement has appeared so it is plain that this condition should be fulfilled. The penalty if this condition is not fulfilled is a marked decrease in the power received from signal waves with no change in the energy received from interferent sources, if the coupling is left constant. Or if the coupling is increased to bring the power from the signal waves back to the maximum value, the energy received from interferent sources increases at the same time. In either case the selective coefficient is diminished. Therefore this adjustment is an important one.

The value of the constant T_{c1} does not affect the maximum power obtainable and neither does it affect to any noticeable extent the selective coefficient against strays. It comes into the discussion only when the steady state selective coefficient against detuned frequencies is being considered. The approximate expressions for this coefficient as given in (41c) is,

$$S_c \text{ (against detuned frequencies)} = \frac{p^4\omega^4 T_{c1}^2 T_{c2}^2}{(1+J)^2} \qquad (41c)$$

Now if T_c and J are constant, the sum of T_{c1} and T_{c2} must be approximately constant from equation (48). Then the largest value of the product T_{c1} T_{c2} is obtained when T_{c1}, and T_{c2} are about equal. Therefore this is the condition toward which T_{c1} should be adjusted. This means giving T_{c1} values of the order of .01. Large values are obtained by building small antennas, or loop aerials in which wasteful resistance are carefully reduced. The feasibility of obtaining antenna circuits with time constants as long as .01 seconds, was discussed at more length in the previous paper and it was snown that such values can be obtained. The penalty if this condition is not fulfilled is not extremely severe since the selective coefficient against strays is practically unaffected by the value of T_{c1}. But after T_{c1} falls below one tenth of T_{c2}, the selective coefficient against detuned frequencies varies practically as T_{c1}^2, since all other factors in (41c) are then nearly constant. Therefore for the sake of large values of this selective coefficient, T_{c1} should approach as closely as possible to the value of T_{c2} .

When $T_{c1} = T_{c2}$ equation (48) gives the relation that each is equal to 0.4 T_c $(1+J)$ and substituting these values in (41c) gives

$$S_c \text{ (against detuned frequencies)} = \frac{p^4\omega^4\left[.4\, T_c^2(1+J)^2\right]^2}{(1+J)^2}$$

$$= 0.2\, p^4\omega^4 T_c^4(1+J)^2 \qquad (41d)$$

Or when T_{c1}/T_{c2} is equal to any constant, both T_{c1} and T_{c2} are proportional to $T_c \cdot (1+J)$, and the above expression holds except for the numerical coefficient.

This equation states the selective coefficient against detuned frequencies in terms of the two constants remaining to

be considered, T_c and J.

From the equation it is seen that this selective coefficient varies as T_c^4 from which it appears that speed of sending might be sacrificed in order to increase T_c as far as possible; even beyond .01 second possibly. But the interference from strays is the most troublesome question and equation(60) shows that the selective coefficient against strays varies only as the first power of T_c. Therefore even if it were possible to double T_c, and if the sending speed were reduced one half, the selective coefficient against strays would be only four times as great. The conclusion is therefore that T_c should be made as large as .01 sec. but that probably no increase beyond .01 sec. would be warranted.

The next constant to be considered is the coupling index J. When the other six independent constants have been fixed as suggested, the value of this index will be determined by effecting a compromise between the requirement for the maximum power from the signal and those for a maximum selective coefficient against sustained detuned frequencies. The selective coefficient against strays was found to be independent of this constant.

When the reactances X_1 and X_2 are zero, the relation between the power delivered to the secondary, and the coupling index, are given by equation (34c) previously derived

$$P_2 = \frac{E_1^2 J}{R_1(1+J)} \qquad (34c)$$

The maximum power is delivered to the detector when J = 1. The power varies only 20% however for values of J varying from .4 to 1.6 as may be seen from Fig. 16, in which the relation (34c) between P_2 and J is shown graphically.

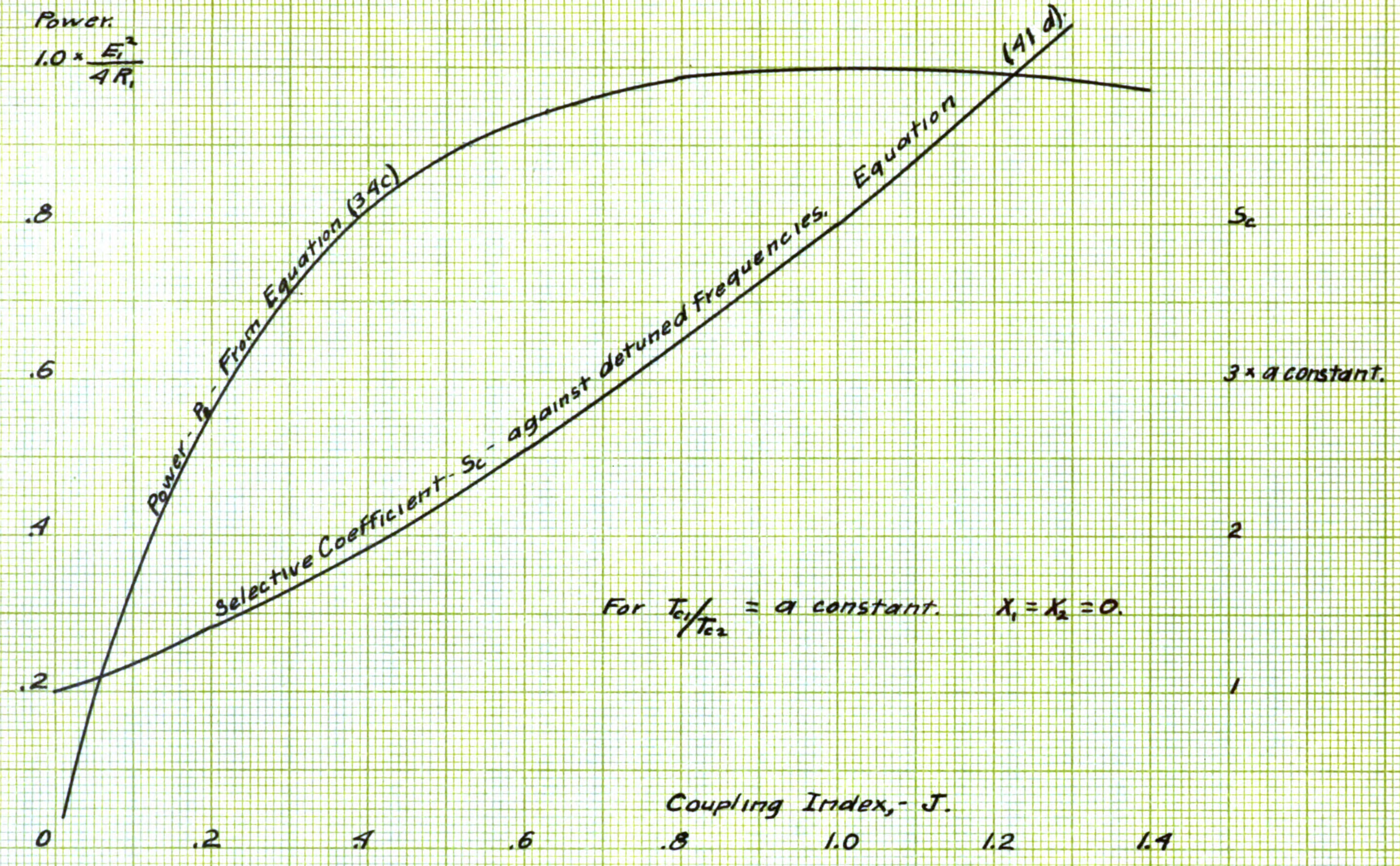

Power.
$1.0 \times \frac{E_1^2}{4R_1}$
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
1.2
1.4
Power - P_2 - From Equation (34c)
Selective Coefficient - S_c - against detuned frequencies. Equation (41 d).
S_c
3 × a constant.
2
1
For T_{c1}/T_{c2} = a constant. $X_1 = X_2 = 0$.
Coupling Index, - J.

Fig.- 16.

The relation between the selective coefficient against detuned frequencies, and the coupling index is given in equation (41d). This equation states the relation between S_c and J when T_c is assumed constant, and T_{c1}/T_{c2} is equal to one, or at least equal to a constant. These are the relations which have been chosen in this section so (41d) is the equation which should be used. This equation is also shown graphically in Fig. 16.

Caution must be used in drawing conclusions from this equation or its graph. The selective coefficient is shown increasing as $(1+J)^2$, so the choice naturally would be swung toward large values of J. But in deriving equation (41d) it was assumed that T_{c1} and T_{c2} each equal $0.4T_c(1+J)$. When $T_c = .01$ and $J = 1$, the value of T_{c1} or T_{c2} is almost .01. Therefore it is doubtful if equation (41d) can be made to hold when J is much larger than one. And in the next section in which the effect of the unavoidable variations in sending frequency are considered, further reasons are found why J should not be made larger than one. Thus the decision is reached that $J = 1$ is the best value to use.

And R_1 is the only constant left to choose. It will be fixed by means of the principles stated in the preceding paper. An antenna with $X_1 = 0$ and $T_{c1} = .008$ can be built as shown there, with almost any dimensions desired. But as the dimensions are increased the ratio of the radiation resistance to the wasteful resistance will increase also. Or the abstractive efficiency will increase. The choice of the antenna dimensions is finally obtained as a balance between cost considerations and efficiency considerations.

But as mentioned previously an amplifier can be used instead of increasing the abstractive efficiency. So usually only a moderate cost of antenna would be justified. That is, the antenna would be rather small, and the desired time constant would be secured by letting the wasteful resistances be relatively large. And power amplifiers would be used to increase the signal strength.

When the antenna dimensions are chosen all the circuit constants are fixed and the problem of choosing the receiving circuit constants is finished except for the study made in the next section to see if the unavoidable variations in sending frequency require the values of any constants to be varied from the values herein chosen.

The coupled receiving circuit then which would have the best operating characteristics for a given frequency would be laid out with the following constants.

R_2 is the equivalent series resistance of the detector.

R_1 is fixed by choosing the antenna dimensions so as to get the largest abstractive efficiency possible at a reasonable cost.

$X_1 = X_2 = 0$

$J = 1$ or $X_m^2 = R_1 R_2$

$T_c = .01$

And then from equation (48) and assuming that $T_{c1} = T_{c2}$

$T_{c1} = T_{c2} = .0083$.

This whole study has been concerned with the problem of choosing the constants for the complete station, antenna circuit as well as the rest of the circuit. But it may be well to

briefly draw conclusions for another set of conditions. Assume that the antenna circuit has been extablished that is, R_1 and T_{c1} have been fixed. How should the other constants be chosen?

The values of T_c and J should be chosen the same in this case as before, so that T_{c2} would be fixed by equation (48) and practically the only difference in the two cases would lie in the values of T_{c1} and T_{c2}. T_{c1} probably would not be equal to the value .008 chosen above. And then T_{c2} would also differ since it is adjusted to satisfy equation (48). If T_{c1} was very short, T_{c2} would approach .02 in value.

14. Effect in the Receiving Circuit of unavoidable Variations in tuning Adjustments and in sending Frequency.

In the study of simple receiving circuits, the conclusion was reached that no increase in the selectivity could be obtained by increasing the time constant of the circuit beyond a certain limit, because of the unavoidable variations in sending frequency. With coupled circuits, the effect of the same unavoidable variations in frequency must now be considered to see if the limit which they place upon the time constant T_c, is below the value already chosen, .01 seconds.

There are two separate possibilities to be considered. First; the two receiving circuits may have the same resonant frequency but the sending station a slightly different frequency, and second; the two receiving circuits may differ slightly in resonant frequency while the sending frequency has yet another value. The first possibility will be discussed first. The conclusions will be drawn from a "resonance curve" or a curve showing the relation between the current in the secondary

circuit and the number of cycles per second of detuning. This curve is shown in Fig. 17 as Curve A.

The equation of this curve is obtained from equation (33). The circuit constants are assumed to be exactly as chosen in the preceding section, except that X_1 and X_2 are not exactly equal to zero because of the variation in sending frequency.

$$I_2 = \frac{E_1 X_m}{\left[X_m^4 + 2X_m^2(R_1R_2 - X_1X_2) + (R_1^2 + X_1^2)(R_2^2 + X_2^2)\right]^{1/2}} \tag{33}$$

To make numerical calculations easier, substitutions are made as follows: $X_1 = p\omega R_1 T_{c1}$, $X_2 = p\omega R_2 T_{c2}$, and $X_m^2 = J R_1 R_2$ Then upon collecting important terms, I_2 is given by,

$$I_2 = \frac{E_1}{2R_1\left[1 + (.045pf)^4\right]^{1/2}} \tag{33a}$$

in which p represents as before, the decimal parts detuning between sending and receiving stations. Or the sending frequency is equal to $(1+p)$ times the resonant frequency of the receiving circuits. And pf gives the detuning in cycles per second.

Now calculations can be made in a similar way for the other conditions in which the two receiving circuits have different resonant frequencies. The curve B in Fig. 17 is drawn for a difference of 15 cycles per second between these frequencies. At 30,000 signal frequency this amounts to .05%. As is seen the effect of this is to lower the flat part of the curve about 30%. The rest of the curve is practically the same as before. Or the effect of departing from the condition $X_1 = X_2 = 0$, is, as has been stated before, to reduce the selective coefficient.

One would expect the two circuits to be held together more

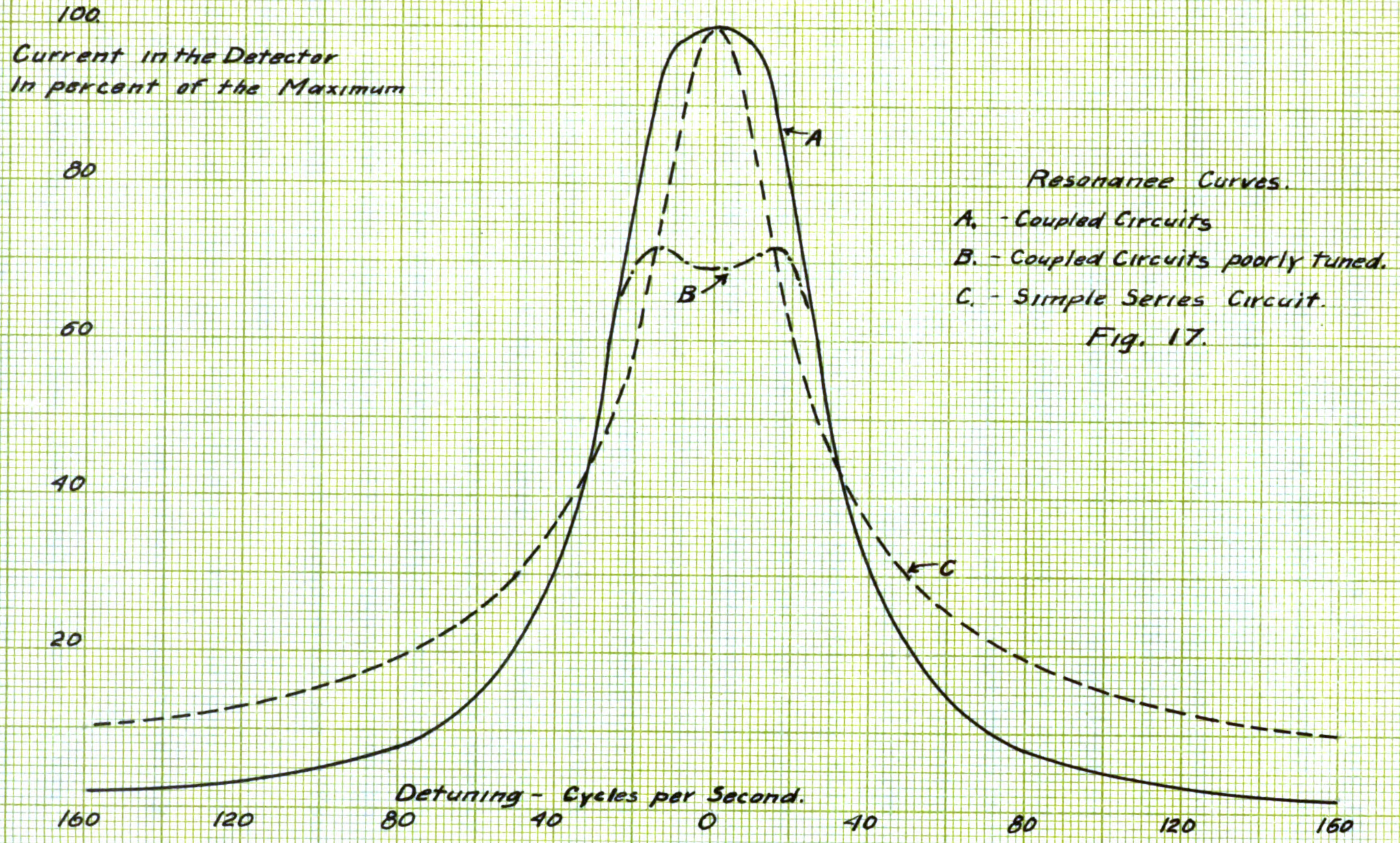

100
Current in the Detector
In percent of the Maximum
80
60
40
20
A
B
C
Resonanee Curves.
A. - Coupled Circuits
B. - Coupled Circuits poorly tuned.
C. - Simple Series Circuit.
Detuning - Cycles per Second.
160
120
80
40
0
40
80
120
160

Fig. 17.

closely than is assumed for this curve B since the power obtained from the signal waves under these conditions is only one half of the maximum power, and therefore the better adjustments could be very easily distinguished.

For comparison with these curves, is shown the resonance curve "C" for a simple series circuit having the same time constant, .01 seconds. This comparison shows an advantage of the coupled circuit in that the resonance curve for the coupled circuits is much flatter and broader on top but drops much more rapidly and falls below the other for all detuning greater than 30 cycles per second. This is a much more desirable curve.

Defining the allowable detuning as that detuning which reduces the current to 70% of the maximum value, or the power to 1/2, we see from the curve that the allowable detuning is about 23 cycles per second. If the sending frequency is 23,000 or the wave length 13,000 meters, this allowable detuning amounts to 0.1%. It is understood that high frequency alternators can be held at speeds constant within these limits, and it would seem that in arc generators the limits could be more easily attained. It will be assumed here that this limit is attainable in large stations and that the circuit constants chosen in the preceding section need not be modified because of the effect of these variations in frequency.

If these limits are not attainable, the time constant T_c, must be reduced until the limits are extended sufficiently to be attainable.

The allowable detuning when the coupled circuits are used is 50% greater than when simple series circuits are used. This

is another way of stating the advantage of coupled circuits. It is beyond the purpose of this paper to investigate the properties of other special circuits but it may be suggested that by using three or four tuned circuits coupled chain fashion, the resonance curve could be given even a broader top, and steeper sides than in Fig. 17. That is, the allowable detuning would be greater, and the selective coefficients against frequencies detuned 1 or 2% would be greater than before. This statement checks with the known properties of "current filters" which are used extensively in telephony.

In the previous section the coupling index was chosen as 1, altho variations between $\frac{1}{2}$ and 2 apparently made no great difference. In this section additional reasons are presented why J should be very close to 1. Curves similar to curve A. Fig. 17, but corresponding to values of J less than one, have peaks becoming more sharp and pointed, therefore the allowable detuning is less and the circuit is less desirable. And when J is made greater than one, the curve has two peaks, each one sharper than the peak of curve A, Fig. 17. This again is a less desirable condition. The curve A, approaches quite closely to the ideal resonance curve, and the strongest argument in favor of making J exactly equal to one, is that this adjustment corresponds to the curve A.

CHAPTER III

Factors which should govern the Choice of the Operating Frequency.

15. Figure of Merit of the Frequency when Coupled Receiving Circuits are used.

Just as the selective coefficients are useful in comparing the merits of different receiving circuits operating on the same frequency, so a new quantity, the "Figure of Merit of the frequency" which was introduced in the preceding paper, is useful in that it gives a convenient and just basis for comparing the merits of different operating frequencies.

To explain the term again briefly, we may assume that a given sending and receiving station are to operate at a fixed distance apart, and that the sending station is to radiate energy at a given rate, independent of the frequency. And remember that the criterion by which the different frequencies are to be judged is the ratio at the detector between the energy received from the correspondent's signal and the energy received from interferent sources.

This ratio is equal to the selective coefficient S_c when the peak electric intensities of the signal waves and of the interferent waves are equal at the receiving station. But under the conditions assumed above, the electric intensity of the signal waves varies with the frequency because of transmission losses. The power received from these waves is directly proportional to the transmission efficiency T_r. Therefore the ratio between the energies under these conditions, is directly proportional to the product, (Selective coefficient) x (Trans-

mission efficiency), or $S_c T_r$. This product was defined in the previous paper as the "figure of merit of the frequency".

If equation (22), $T_r = \varepsilon^{-\frac{.003r}{\sqrt{\lambda}}}$, is accepted as representing the best information about the transmission efficiency, this product may be written out. The figures of merit of the frequency when the interference is from detuned stations, is from equations (22) and (41d)

$$S_c T_{r\,(\text{detuned frequencies})} = .2 p^4 \omega^4 T_c^4 (1+J)^2 \times \varepsilon^{-\frac{.003r}{\sqrt{\lambda}}}$$

$$= 320\, p^4 T_c^4 (1+J)^2 \cdot f^4 \varepsilon^{-5.5 r\sqrt{f} \times 10^{-5}} \qquad (63)$$

When the interference is from strays, the figure of merit becomes approximately, from (22) and (62)

$$S_c T_{r\,(\text{strays})} = 16\, b T_e T_c f^2 \varepsilon^{-5.5 r\sqrt{f} \times 10^{-5}} \qquad (64)$$

Which one of these expressions is to be used depends upon which type of interference is more important. Usually the strays are more important and (64) would be used.

By use of this figure of merit, the operating frequency which will give the highest ratio between the energy received from signals and that received from interference may be found.

A tabulation of the figure of merit, or of its two factors S_c and T_r for various frequencies and distances of transmission is a convenient form for presenting the conclusions. In Table IX of the previous paper such a tabulation was presented for simple circuits. The transmission efficiencies are the same of course for coupled circuits, and the selective coefficients against strays, is practically the same for coupled circuits, as for simple circuits with the same time constant. Therefore the figure of merit when the important interference is from strays, can be taken from Table IX either for simple or for

coupled circuits.

16. Costs of the radio system.

The only object in building large and costly antennas for receiving is to increase the energy received by the antenna by reason of the better abstractive efficiencies of the larger antennas. But power amplification, accomplishes the same purpose so that we are relieved of the necessity for the costly receiving antennas.

For sending antennas the cost have already been discussed in the previous paper, as far as general statements can be made.

If the important interference is from strays, then, both the figure of merit of the frequency and the costs of the system are the same for coupled as for simple receiving circuits, and the choice of operating frequency does not depend upon the type of receiving circuit used. The conclusions of the previous paper hold for coupled circuits also.

But if the important interference is from detuned stations, the advantage of high frequencies is much greater with coupled circuits than with simple circuits, because the coupled circuit selective coefficient varies as the fourth power of the frequency instead of the second. In this case the choice of operating frequency would depend on whether simple or coupled receiving circuits were to be used. And with coupled circuits probably higher frequencies would be chosen.

It is not the purpose in this paper to deal more in detail with the subjects of costs. Cost estimates must be prepared for each special case, and the general problem has been carried to the point where a tabulation of cost estimates at the different

frequencies, and of the figures of merit of the frequencies, will provide a basis for an intelligent choice of the operating frequency.

CHAPTER IV

Conclusions Drawn from this Study

17. Comparison of Coupled with simple Series receiving Circuits.

Coupled circuits or any other special circuits are used usually in the hope of obtaining greater selective properties. But in comparing these circuits with simple circuits they should be compared in other respects also, such as power supplied to the detector, allowable variations in sending frequency, etc. Most of these comparisons between simple circuits and coupled circuits have already been made in the various sections of this paper, so only a summary is necessary here.

The maximum power in the secondary of a coupled circuits receiving station is the same as the maximum power in the detector in a simple circuit provided the abstractive efficiencies are the same. And therefore except for the power losses in the secondary circuit the two stations are equivalent in this respect. The secondary power losses can be kept within 10% of the total secondary power by using tuning coils in which $2L/R = 0.1$. On the whole there is little difference between the two circuits with respect to power in the detector.

The limit placed upon the time constant by the requirement that there shall be an interval of silence between tones in the receiver, is the same for both circuits. But the allowable detuning is 50% greater for coupled circuits than for a simple circuit with the same time constant, so that the limit placed upon the time constant by the unavoidable variations in frequency is higher for coupled circuits than for simple circuits. At operating frequencies of 30,000 or above, this

advantage of coupled circuits becomes quite great, since then the time constant may be determined largely by the effect of unavoidable variations in frequency.

The steady state selective coefficient against detuned frequencies is much larger for coupled circuits than for simple circuits. Comparison should be made between equation (7) for simple circuits and equation (41d) for coupled circuits. The time constants should be the same in both for a just comparison, and therefore we have

$$S_c \text{ (coupled circuits)} \doteq 0.2\, S_c^2 \text{ (simple circuit)} \qquad (41e)$$

Thus it is evident that a very great increase in this selective coefficient is possible by using coupled circuits instead of simple circuits. When interference from detuned stations is an important problem, the use of two or even more coupled circuits, offers great advantages. The advantage in this respect is another big advantage of coupled circuits over simple circuits.

In comparing the selective coefficients of the two circuits against strays the time constants should again be considered equal. And under these conditions the selective coefficients in the two cases are equal, at least for all practical purposes. Nothing is gained then in the way of greater selectivity against strays by using coupled circuits.

But no doubt many have observed that on particular antennas the interference from strays can be greatly reduced by using a tuned secondary circuit, and they will ask to have the above statement checked with this fact. The answer is that these observations are made on antenna circuits with very short time constants. Consider for example the Darien Antenna

at 120,000 cycles per second. With a detector in the primary the time constant of the simple receiving circuit becomes .00001 seconds, while with even the most ordinary tuning coil for a secondary circuit the time constant of the coupled circuit will be of the order of .001 sec. or 100 times as long. Therefore in this case the selective coefficient against strays should increase 100 to 1 when the secondary circuit is used.

The conclusion then is that if an antenna is to be built and is designed to have a time constant of .01 seconds as a simple receiving circuit, no increase can be made in the selective coefficient against strays by using a secondary circuit with this antenna. But if an antenna circuit with a short time constant is to be used a big gain can be made by used coupled receiving circuits.

18. Details in which the proposed Circuits may Differ from modern practice.

The detail in which the suggested circuit differs most widely from most modern practice is probably in the length of the time constants. No published data has been found in which the values 2L/R for either primary or secondary circuit approached the value .01.

It has been the endeavor in this paper to determine quantitatively just what increase in selective coefficients might be expected when the time constants are increased say from .001 to .01. In this case the selective coefficient against detuned frequencies would be 10,000 times as great and the coefficient against strays would be 10 times as great. This gain should justify considerable trouble and expense.

Coils with long time constants are obtained by using large coils with increased weight of copper, or by means of the resistance neutralization, which is possible with three electrode vacuum bulbs. Antenna circuits with long time constants are obtained by building small antennas and carefully reducing or neutralizing a part of the wasteful resistance.

Another detail in which this treatment suggests a departure from a large part of the modern practice, is in adjustment of the coupling. Altho loose coupling has been often suggested as a means of increasing the selectivity no definite idea can be gathered as to what limit this is carried in practice. But it seems probable that in a large number of cases the circuits as used would give a value of the coupling index much larger than 1. The advantage of decreasing J to 1 is brought out in Table XI and resonance curve A, Fig. 17.

APPENDIX A

The Transient Currents in Coupled Circuits when $T_{c1} = T_{c2}$.

The purpose in this appendix is to outline the steps in the derivation of the general expressions for the transient currents in two coupled circuits, in which $T_{c1} = T_{c2}$ and $1/L_1C_c = 1/L_2C_2 = \omega^2$. This circuit is one in which the general solution can be carried out completely since the fourth degree equation which appears, can be solved exactly in literal form. For other circuits in general this is not true, the solution must be obtained approximately in numerical form.

In dealing with the differential equations of the circuits the coefficient of coupling, $k = M/\sqrt{L_1L_2}$, will be used. It has appeared but little in the previous sections of this paper. In terms of the "coupling index" J, the coefficient of coupling becomes for this case,

$$\omega k = \frac{\omega M}{\sqrt{L_1L_2}} = \frac{X_m}{\sqrt{L_1L_2}} = 2\sqrt{\frac{J\, R_1 \quad R_1}{(2L_1)(2L_2)}} = \frac{2\sqrt{J}}{T_{c1}} \tag{66}$$

The differential equations applying to two coupled circuits have been already given, in equations (44). In this case e_1 is assumed to be constant so $\frac{de_1}{dt} = 0$. Then

$$\begin{aligned} L_1\frac{d^2 i_1}{dt^2} + R_1\frac{d i_1}{dt} + \frac{i_1}{C_1} + M\frac{d^2 i_2}{dt^2} &= 0 \\ L_2\frac{d^2 i_2}{dt^2} + R_2\frac{d i_2}{dt} + \frac{i_2}{C_2} + M\frac{d^2 i_1}{dt^2} &= 0 \end{aligned} \tag{44}$$

Assume that the currents in the two circuits are expressed by terms of the form $\varepsilon^{\gamma t}$, that is $i_1 = A\varepsilon^{\gamma t}$ and $i_2 = B\varepsilon^{\gamma t}$

Substituting these values in (44) and dividing thru by L_1 and L_2 respectively gives,

$$\begin{aligned} \left(\gamma^2 + \frac{R_1\gamma}{L_1} + \frac{1}{L_1C_1}\right)A\varepsilon^{\gamma t} + \frac{M\gamma^2}{L_1}B\varepsilon^{\gamma t} &= 0 \\ \left(\gamma^2 + \frac{R_2\gamma}{L_2} + \frac{1}{L_2C_2}\right)B\varepsilon^{\gamma t} + \frac{M\gamma^2}{L_2}A\varepsilon^{\gamma t} &= 0 \end{aligned} \tag{67}$$

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$$\omega k = \frac{\omega M}{\sqrt{L_1 L_2}} = \frac{X_m}{\sqrt{L_1 L_2}} = 2\sqrt{\frac{J\, R_1 \quad R_2}{(2L_1)(2L_2)}} = \frac{2\sqrt{J}}{T_{c1}} \tag{66}$$

The differential equations applying to two coupled circuits have been already given, in equations (44). In this case e_1 is assumed to be constant so $\frac{de_1}{dt} = 0$. Then

$$\begin{aligned} L_1\frac{d^2 i_1}{dt^2} + R_1\frac{di_1}{dt} + \frac{i_1}{C_1} + M\frac{d^2 i_2}{dt^2} &= 0 \\ L_2\frac{d^2 i_2}{dt^2} + R_2\frac{di_2}{dt} + \frac{i_2}{C_2} + M\frac{d^2 i_1}{dt^2} &= 0 \end{aligned} \tag{44}$$

Assume that the currents in the two circuits are expressed by terms of the form $\varepsilon^{\gamma t}$, that is $i_1 = A\varepsilon^{\gamma t}$ and $i_2 = B\varepsilon^{\gamma t}$

Substituting these values in (44) and dividing thru by L_1 and L_2 respectively gives,

$$\begin{aligned} \left(\gamma^2 + \frac{R_1\gamma}{L_1} + \frac{1}{L_1C_1}\right)A\varepsilon^{\gamma t} + \frac{M\gamma^2}{L_1}B\varepsilon^{\gamma t} &= 0 \\ \left(\gamma^2 + \frac{R_2\gamma}{L_2} + \frac{1}{L_2C_2}\right)B\varepsilon^{\gamma t} + \frac{M\gamma^2}{L_2}A\varepsilon^{\gamma t} &= 0 \end{aligned} \tag{67}$$

Solving these equations for $A\varepsilon^{\gamma t}$, gives,

$$A\varepsilon^{\gamma t}\left[\left(\gamma^2+\frac{2\gamma}{T_{c1}}+\omega^2\right)\left(\gamma^2+\frac{2\gamma}{T_{c2}}+\omega^2\right)-\frac{M^2\gamma^4}{L_1L_2}\right]=0$$

If the coefficient A is to be different from zero, the expression in the bracket must equal zero. Since $T_{c1} = T_{c2}$ this is written

$$\left(\gamma^2+\frac{2\gamma}{T_{c1}}+\omega^2\right)^2-k^2\gamma^4=0 \tag{68}$$

This equation is of the fourth degree in γ but may be reduced by factoring to the two equations,

$$\gamma^2+\frac{2\gamma}{T_{c1}}+\omega^2+k\gamma^2=0$$
$$\gamma^2+\frac{2\gamma}{T_{c1}}+\omega^2-k\gamma^2=0$$

Solving these two equations for γ gives the four roots of (68)

$$\gamma=-\frac{1}{T_{c1}(1+k)}\pm j\sqrt{\frac{\omega^2}{1+k}-\frac{1}{T_{c1}^2(1+k)}}$$
$$\gamma=-\frac{1}{T_{c1}(1-k)}\pm j\sqrt{\frac{\omega^2}{1-k}-\frac{1}{T_{c1}^2(1-k)}}$$

The first pair of roots will be represented by $\alpha_1 \pm j\beta_1$ and the second pair by $\alpha_2 \pm j\beta_2$

Thus there are four values of γ which give possible terms in the expression for the current, i. e.,

$$\begin{aligned} i_1&=A_1\varepsilon^{(\alpha_1+j\beta_1)t}+A_2\varepsilon^{(\alpha_1-j\beta_1)t}+A_3\varepsilon^{(\alpha_2+j\beta_2)t}+A_4\varepsilon^{(\alpha_2-j\beta_2)t}. \\ i_2&=B_1\varepsilon^{(\alpha_1+j\beta_1)t}+B_2\varepsilon^{(\alpha_1-j\beta_1)t}+B_3\varepsilon^{(\alpha_2+j\beta_2)t}+B_4\varepsilon^{(\alpha_2-j\beta_2)t}\end{aligned} \tag{69}$$

This is known to be mathematically equivalent to

$$\begin{aligned} i_1&=A_1'\varepsilon^{\alpha_1t}\cos\beta_1t+A_2'\varepsilon^{\alpha_1t}\sin\beta_1t+A_3'\varepsilon^{\alpha_2t}\cos\beta_2t+A_4'\varepsilon^{\alpha_2t}\sin\beta_2t. \\ i_2&=B_1'\varepsilon^{\alpha_1t}\cos\beta_1t+B_2'\varepsilon^{\alpha_1t}\sin\beta_1t+B_3'\varepsilon^{\alpha_2t}\cos\beta_2t+B_4'\varepsilon^{\alpha_2t}\sin\beta_2t.\end{aligned} \tag{70}$$

in which A_1', A_2', B_1', B_2' etc. represent undetermined constants. The equivalent constants in equation (69) are A_1, A_2, B_1, B_2 etc.

The eight coefficients A_1', A_2', B_1', B_2' etc. are determined

by making equation (70) fulfill several necessary conditions. The ratios between A_1' and B_1', A_2' and B_2' etc. are fixed by substituting (70) into the differential equations. The remaining constants are fixed by making the equation (70) give the known values of current in the two circuits at the instant the voltage was applied, and by deriving from (70) the expressions for the condenser charges or the condenser voltages, and making these expressions fit the known values at the instant the voltage was applied. As a result of these determinations the exact expressions for the currents when a steady voltage E_1 is suddenly applied to the primary, are

$$i_1 = \frac{E_1}{2\beta_1 L_1(1+k)} \varepsilon^{\alpha_1 t} \sin\beta_1 t + \frac{E_1}{2\beta_2 L_1(1-k)} \varepsilon^{\alpha_2 t} \sin\beta_2 t$$

$$i_2 = \sqrt{\frac{L_1}{L_2}} \left[\frac{E_1}{2\beta_1 L_1(1+k)} \varepsilon^{\alpha_1 t} \sin\beta_1 t - \frac{}{2\beta_2 L_1(1-k)} \varepsilon^{\alpha_2 t} \sin\beta_2 t \right] \qquad (71)$$

These expressions involve no approximations as they stand, but in order to arrive at short expressions for the power and energy, some approximations are introduced at this point.

For the circuits under consideration in this paper, β_1 can be written very closely as $\sqrt{\omega^2/(1+k)}$ since α_1^2 is small compared to ω^2. And then this is very closely equal to $\omega(1-k/2)$ since k is so small compared to unity.

And in the expression for either i_1 or i_2 the denominators of the coefficients of the two terms differ by very small amounts and $2\beta_1 L_1(1+k) \approx 2\beta_2 L_1(1-k) = 2\omega L_1$ approximately. The damping factors $\varepsilon^{\alpha_1 t}$ and $\varepsilon^{\alpha_2 t}$ are each very closely equal to $\varepsilon^{-t/T_{c1}}$. Making these further substitutions the expression for i_1 becomes

$$i_1 = \frac{E_1}{2\omega L_1} \varepsilon^{-t/T_{c1}} \left[\sin\left(\omega t - \frac{k\omega t}{2}\right) + \sin\left(\omega t + \frac{k\omega t}{2}\right) \right]$$

By a trigonometric transformation this becomes

$$i_1 = \frac{E_1}{\omega L_1} \varepsilon^{-t/T_{c1}} \cos\frac{k\omega t}{2} \cdot \sin\omega t.$$

The peak value of the current in any cycle is given by $\frac{E_1}{\omega L_1} \varepsilon^{-t/T_{c1}} \cos\frac{k\omega t}{2}$, and the average power over the cycle is

$$P_1 = \frac{1}{2} R_1\, i^2_{1(peak)} = \frac{R_1 E_1^2}{4\omega^2 L_1^2}\, \varepsilon^{-2t/T_{c1}} (1 + \cos k\omega t)$$

From (65) $2\sqrt{J}/T_{c1}$ can be substituted for $k\omega$.

The total energy expended in this circuit is given by

$$\int_0^\infty P_1\, dt = \frac{R_1 E_1^2}{4\omega^2 L_1^2} \int_0^\infty \varepsilon^{-2t/T_{c1}} \left(1 + \cos\frac{2\sqrt{J}\, t}{T_{c1}}\right) dt$$

$$\frac{R_1 E_1^2}{4\omega^2 L_1^2} \cdot \frac{T_{c1}(2+J)}{2(1+J)} = \frac{C_1 E_1^2 (2+J)}{4(1+J)}$$

By similar steps the expression for i_2 becomes

$$i_2 = \sqrt{\frac{L_1}{L_2}} \cdot \frac{E_1}{\omega L_1} \varepsilon^{-t/T_{c1}} \sin\frac{\sqrt{J}\, t}{T_{c1}} \cdot \cos\omega t$$

And $$P_2 = \frac{R_1 L_1}{L_2} \frac{E_1^2}{4\omega^2 L_1^2} \varepsilon^{-2t/T_{c1}} \left(1 - \cos\frac{2\sqrt{J}\, t}{T_{c1}}\right) = \frac{R_1 E_1^2}{4\omega^2 L_1^2} \varepsilon^{-2t/T_{c1}} \left(1 - \cos\frac{2\sqrt{J}\, t}{T_{c1}}\right).$$

and the energy dissipated in the secondary is given by

$$W_2 = \frac{E_1^2 J}{4\omega^2 L_1 (1+J)} = \frac{C_1 E_1^2 J}{4(1+J)}$$

APPENDIX B

A Method of Tuning which will lead to the Desired Relations, $X_1 = X_2 = 0$, and $X_m^2 = R_1 R_2$.

When tuning adjustments are governed by the sounds in the receiver, the practice in a large majority of cases is to adjust for maximum response to the signals or maximum power in the detector. This is an easy adjustment to make. But it was pointed out in section 9 that maximum power could be obtained with many combinations in which X_1 and X_2 differ from zero, and in fact if the circuits are tuned for maximums power alone, it is very improbable that the condition, $X_1 = X_2 = 0$, is obtained. The figures given below will illustrate this point. Therefore in tuning it is evident that something is necessary besides tuning for maximum power, and the purpose of this appendix is to outline a method of adjusting the circuits which will lead to the desired relations $X_1 = X_2 = 0$, and $J = 1$ or $X_m^2 = R_1 R_2$.

To illustrate the proposed method concretely, it will be assumed that the circuit constants are as specified in section 13. Since R_1 and R_2 are still arbitrary we will assume $R_1 = R_2 = 1$ for ease in calculation. Further assume that it is desired to adjust the circuit to receive at a frequency of 30,000 (or a wave length of 10,000 meters) and assume that the secondary circuit happens to be detuned 20%. Then $X_1 = p\,\omega R_1 T_{c1}$ $= .20 \times 188000 \times .0083 = 320$. (See equation (42))

Now let the first adjustments be made on X_m and X_1. X_m should be set at various values above $X_m^2 = R_1 R_2 = 1$ and X_1 should be varied in an effort to pick up the signal. With $X_2 = 320$, from equations (35) and (36) the values of X_m and X_1

which give maximum power in the detector are $X_m = 320$, and $X_1 = 320$.

Assume further that the signals become audible when the power is 10% of the maximum power or when the current in the detector is about 1/3 of the maximum. Now if X_m is set by trial at any value above 40, calculations show that the signals will become audible as X_1 passes thru the value specified in equation (35). For example, when X_m is set for trial at 40, equation (35) shows that the value of X_1 which gives largest power is $320 \times 40^2/(1+320) = 5$. As X_1 passes thru this value, substitution of numerical values in equation (34) shows that $P_2 = 10\%$ of $E_1^2/4R_1$, or the signals will become audible. And if X_m happens to be set for trial at 100, when X_1 passes through the value, 31, P_2 becomes 30% of $E_1^2/4R_1$. Thus for these circuits it should not be difficult to pick up the signals at a given wave length, if the constants of the circuit are known even approximately. For ease in picking up signals, the secondary circuit should be detuned from 10 to 40%.

Then after the signals have been picked up, let the operations be as follows: Leave X_2 fixed, and gradually reduce X_m, at the same time keeping X_1 adjusted so as to get as loud signals as possible.

As X_m approaches 40, the signals become weaker, until the assumed limit of audibility is reached when $X_m = 40$, and $X_1 = 5$.

Then leave X_1 fixed and do the same thing with X_m and X_2, that is, decrease X_m and adjust X_2 always to keep the signal as loud as possible. This means that X_2 at any time will have the value given by equation (39). At first the signal strength will increase as X_m decreases until at the values $X_m = 6$, and $X_2 = 5$

the secondary power will have the maximum value, $E_1^2/4R_1$, as may be seen by substituting in equation (34). Then as the operation is continued, the signals will grow weaker again until at the values $X_m = .4$ and $X_2 = .03$, the assumed limit of audibility is again reached.

Repeat this operation alternately with the primary and the secondary until no further reduction can be made in X_m. This will happen in this present case after the next trial. This next operation with X_m and X_1 will leave $X_m = .2$ and $X_1 = .001$. Now the reactances have been reduced as far as is practicable, and in fact until they are negligible in so far as their effect upon signal strength is concerned. The final step is to increase X_m until maximum signal strength is obtained which will occur at $X_m^2 = R_1 R_2$ or $J = 1$.

Thus the desired adjustments have been obtained within only a few operations, and no switching or changing of the circuits is necessary.

Other methods of adjustment may be even more simple, but one main object in presenting this appendix is to emphasize the fact that <u>the best adjustments of coupled receiving circuits are not obtained by simply tuning the circuits for maximum signal strength.</u>